Here's a gnarly integral:

$$\int_{-\infty}^{\infty} \frac{x^4}{64+x^6}\,dx = \int_{-\infty}^{\infty} \frac{x^4}{\left(x^2+4\right)\left(x^4-4x^2+16\right)}\,dx = \int_{-\infty}^{\infty} \frac{x^4}{\left(x^2+16\right)\left(x^2-2\sqrt{3}x+4\right)\left(x^2+2\sqrt{3}x+4\right)}\,dx$$

The partial fractions expansion of this can start this way: $\frac{1}{3}\int_{-\infty}^{\infty} \frac{1}{x^2+16}\,dx + \frac{2}{3}\int_{-\infty}^{\infty} \frac{x^2-2}{x^4-4x^2+16}\,dx$

whence the first integral is $\frac{1}{3}\int_{-\infty}^{\infty} \frac{1}{x^2+16}\,dx = \frac{1}{3}\int_{-\infty}^{\infty} \frac{4}{16u^2+16}\,du =$

$$\frac{1}{12}\arctan\left(\frac{x}{4}\right)\Bigg|_{-\infty}^{0} + \frac{1}{12}\arctan\left(\frac{x}{4}\right)\Bigg|_{0}^{\infty} = 0 - \lim_{b\to-\infty}\frac{1}{12}\arctan\left(\frac{b}{4}\right) + \lim_{b\to\infty}\frac{1}{12}\arctan\left(\frac{b}{4}\right) - 0 = \frac{\pi}{12}$$

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{x^2-2}{x^4-4x^2+16}\,dx &= \frac{1}{8}\int_{-\infty}^{\infty} \frac{\sqrt{3}x-2}{x^2-2\sqrt{3}x+4}\,dx - \frac{1}{8}\int_{-\infty}^{\infty} \frac{\sqrt{3}x+2}{x^2+2\sqrt{3}x+4}\,dx \\
&= \frac{1}{8}\int_{-\infty}^{\infty} \frac{\sqrt{3}x-2}{\left(x-\sqrt{3}\right)^2+1}\,dx - \frac{1}{8}\int_{-\infty}^{\infty} \frac{\sqrt{3}x+2}{\left(x+\sqrt{3}\right)^2+1}\,dx \\
&= \frac{1}{8}\int_{-\infty}^{\infty} \frac{\sqrt{3}u+1}{u^2+1}\,dx - \frac{1}{8}\int_{-\infty}^{\infty} \frac{\sqrt{3}u-1}{u^2+1}\,dx \\
&= \frac{1}{8}\int_{-\infty}^{\infty} \frac{1}{u^2+1}\,dx + \frac{1}{8}\int_{-\infty}^{\infty} \frac{1}{u^2+1}\,dx \\
&= \frac{\pi}{4}
\end{aligned}$$

So the integral is $\frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$