Math 1A – Chapter 1 Test – Sampling of Fair Game - Solutions

1. Consider the line segment connecting the points $\left(2+x^2,2x\right)$ and $\left(3,4x^2\right)$.

   a. Show that the slope of this line segment is given by the function $m\left(x\right)=\dfrac{4x^2-2x}{1-x^2}$.

   A: The slope formula here yields $m=\dfrac{4x^2-2x}{3-\left(2+x^2\right)}=\dfrac{2x\left(2x-1\right)}{1-x^2}=\dfrac{2x\left(2x-1\right)}{\left(1-x\right)\left(1+x\right)}$.

   b. What is the domain of $m(x)$?
   A: The domain will be all x except ±1. In interval notation, $\left(-\infty,-1\right)\cup\left(-1,1\right)\cup\left(1,\infty\right)$.

   c. What are the vertical asymptotes of $m(x)$?
   A: The vertical asymptotes are along $x = \pm 1$.

   d. What is the horizontal asymptote of $m(x)$?
   A: y =2 is the horizontal asymptote.

   e. What are the x-intercepts of $m(x)$?
   A: The x-intercepts are at $x = 0$ and $x = \frac{1}{2}$.

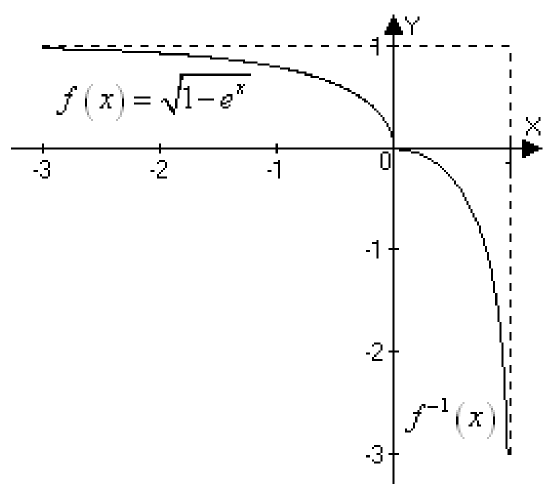
2. Consider the function $f\left(x\right)=\sqrt{1-e^x}$

   a. Find a formula $f^{-1}\left(x\right)$ for the inverse function for f.

   A: $y=\sqrt{1-e^x}\Leftrightarrow y^2=1-e^x;\ y\geq 0\Leftrightarrow x=\ln\left(1-y^2\right);\ y\geq 0$ so $f^{-1}\left(x\right)=\ln\left(1-x^2\right);\ \ x\geq 0$

   b. Complete a short table of values (at least 3 coordinate pairs) and sketch a graph for both $f\left(x\right)$ and $f^{-1}\left(x\right)$ showing the asymptotic behavior of each and their symmetry through the line $y = x$.

x	y
0	0
$\ln\frac{24}{25}\approx -0.041$	$\frac{1}{5}$
$\ln\frac{8}{9}\approx -0.118$	$\frac{1}{3}$
$\ln\frac{3}{4}\approx -0.288$	$\frac{1}{2}$
$\ln\frac{19}{100}\approx -1.66$	$\frac{9}{10}$



3. Consider the function $y(t)=\sin t$.
   a. Write a formula for the function that results from stretching $y(t)$ vertically by a factor 5 and compressing horizontally by a factor 2π.
      A: $f(t)=5y(2\pi t)=5\sin(2\pi t)$
   b. Write a formula for the function that results from stretching and compressing $y(t)$ as in part (a) and then shifting two units up and ¼ unit to the right?
      A: $f(t)=2+5y\left(2\pi\left(t-\frac{1}{4}\right)\right)=2+5\sin\left(2\pi t-\frac{\pi}{2}\right)$

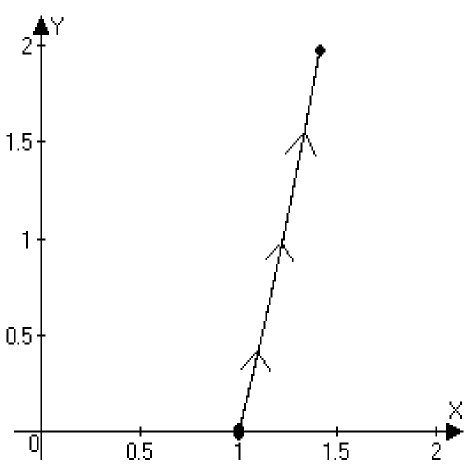
4. An initial population of 12,345 grows by 7% every month.
   a. Find a formula for the population size after *n* months, as a function of *n*.
      A: $P(n)=12345(1.07)^n$
   b. Find the average rate of change of the population over the twelfth week.
      A: $\frac{P(12)-P(11)}{12-11}=12345\left(1.07^{12}-1.07^{11}\right)\approx 1819$ animals per month.

5. Consider the function $g(x)=2x^2-8x-3$.
   a. Write the formula in complete square form; that is, find values for *a*, *h*, and *k* so that $g(x)=a(x-h)^2+k$.
      A: $g(x)=2x^2-8x-3=2\left(x^2-4x\right)-3=2\left(x^2-4x+4\right)-3-8=2(x-2)^2-11$
   b. Describe a sequence of transformations (horizontal/vertical shifts and/or horizontal/vertical stretches and/or reflections) to transform $y=g(x)$ to $y=x^2$.
      A: Shift 11 units up, 2 units left and then compress vertically by a factor 2.

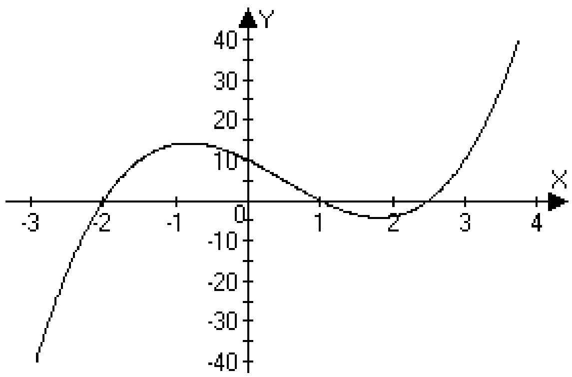
6. Eliminate the parameter for the parametric equations:

$$x=\sec\theta$$
$$y=2\tan^2\theta$$

and sketch a graph for the curve over the interval $0\le\theta\le\frac{\pi}{4}$. Indicate with arrows the direction the curve follows over this interval.
A: From the Pythagorean identity, $\tan^2\theta+1=\sec^2\theta$, we can easily substitute to obtain $\frac{y}{2}+1=x^2 \Leftrightarrow y=2x^2-2$. For $0\le\theta\le\frac{\pi}{4}$, $1\le x\le\sqrt{2}$ and $0\le y\le 2$, following the cuve of the parabola like as shown.



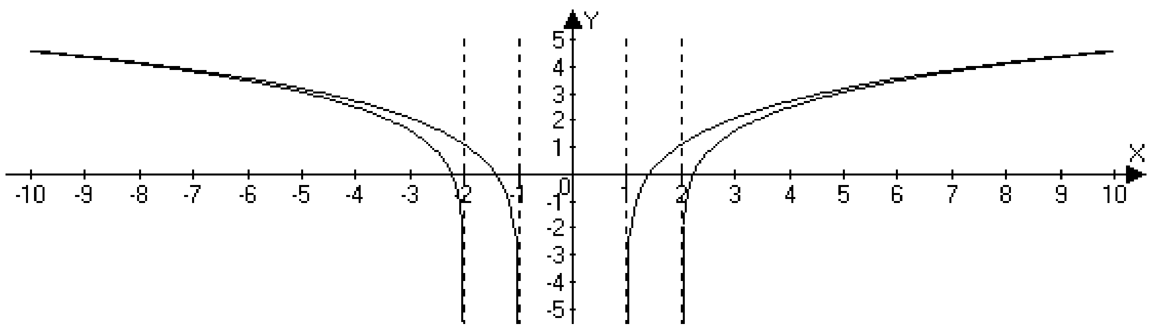
7. Write an equation for the cubic polynomial whose graph is shown



A: Since the roots of the cubic are at x = -2, 1 and 2.5, the cubic will have the form $y = a(x+2)(x-1)(x-2.5)$, to fit the vertical scale, choose a = 2 so the y-intercept fits (0,10): $y = 2(x+2)(x-1)(x-2.5)$

8. Write parametric equations which describe the ellipse with Cartesian equation $\frac{x^2}{2}+\frac{y^2}{4}=1$ and give a range of the parameter which completes one trace of the ellipse.
A: Based on the Pythagorean identity, choose
$\frac{x^2}{2} = \cos^2 t;\ \frac{y^2}{4} = \sin^2 t \Leftarrow \boxed{x = \sqrt{2}\cos t;\ \ y = 2\sin t}.\ \ 0 \le t \le 2\pi$

9. Graph two different members of the family of functions described by $f(x) = \ln(x^2 - c)$ for two different positives values of c. Write a sentence or two describing how the value c shapes the graph.
A: Graphs of $f(x) = \ln(x^2 - 1)$ and $f(x) = \ln(x^2 - 4)$ are shown below. $f(x) = \ln(x^2 - c)$ will have vertical asymptotes along $x = \pm\sqrt{c}$.



10. Consider the function $f(x) = x + 2\ln(x+1)$
   a. Tabulate at least 3 different input/output values for $f(x)$.

A:

x	0	$e-1$	e^2-1
$f(x)$	0	$e+1$	e^2+3

   b. Tabulate at least 3 different input/output values for $f^{-1}(x)$.

A:

x	0	$e+1$	e^2+3
$f^{-1}(x)$	0	$e-1$	e^2-1