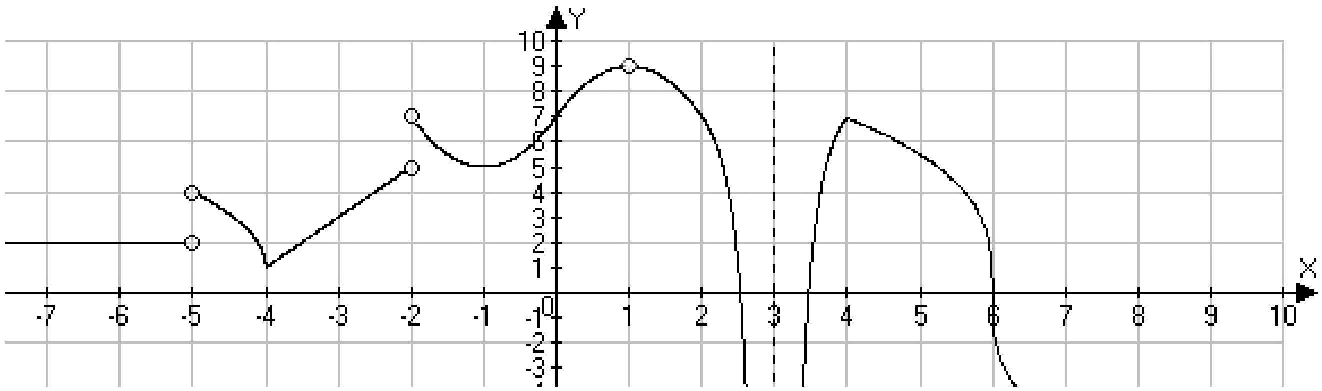
Math 1A – Final Exam – Spring 08 Name____________________

Show your work for credit. Write all responses on separate paper. Do not use a calculator.

1. Consider the following graph for the derivative function $f(x) = F'(x)$. That is, the graph shows the values slopes of the tangent line to $y = F(x)$ for values of x between about –8.7 and 6.7. In what follows, be very careful to distinguish between $F(x), f(x)$ and $f'(x)$.



   a. Note that this tells us nothing about the actual values of $F(x)$, just how $F(x)$ is changing, so further assume that $F(-7) = 0$, what would be the value of $F(-6)$?

   b. Based on the graph, find $\lim_{x \to -5^-} F(x)$ if it exists, if not, explain why not.

   c. Where does $f(x)$ is have a jump discontinuity? List all values of x where this is true.

   d. Where does $f(x)$ is have a removable discontinuity? List all values of x where this is true.

   e. Where does $f'(x)$ have a jump discontinuity?

   f. Over what interval(s) is $F(x)$ increasing?

   g. Where does $F(x)$ have inflection points?

   h. Where is $f(x)$ defined and yet $f'(x)$ is not defined?

2. If the tangent to $y = f(x)$ at (0,2) passes through the point (3,0), find $f'(0)$.

3. Is there a number a such that $\lim_{x \to 1} \frac{x+a}{x^2+x-2}$ exists? If not, why not? If so, find the value of a and the value of the limit.

4. Find the limit. Explain your answers.

   a. $\lim_{x \to 3} \frac{\sqrt{x^2-9}}{x-3}$ b. $\lim_{x \to 0} x \ln x$ c. $\lim_{x \to \infty} \frac{1-\cos x}{x}$

5. Consider the function $f(x)=e^x\cos x$
   a. Find a formula for the second derivative $f''(x)$.
   b. Over what interval(s) is $f(x)$ concave up?

6. Show that $\lim_{u\to 0}\frac{1-\cos u}{u}=0$ and use the definition of the derivative to find $f'(x)$ where $f(x)=\sin x$. You may assume $\lim_{u\to 0}\frac{\sin u}{u}=1$.

7. Suppose a function $y=f(x)$ satisfies the equation $y^2\cos(\pi x)+x^2y+y^3=1$ in a neighborhood of the point (1,1). Find an equation for the tangent line at (1,1).

8. Use a linear approximation to estimate $\arctan\left(\frac{3}{4}\right)$ by considering the tangent line to $y=\arctan x$ at $x=1$. Approximate to 3 significant digits.

9. A spherical balloon is filling with water at a rate of π cm^3/sec. How fast is the surface area increasing when the radius is 2 cm? Useful formulas might include $V=\frac{4}{3}\pi r^3$ and $A=4\pi r^2$.

10. Show there are no values of x in the interval (–1,1) that satisfy the conclusion of the mean value theorem for $f(x)=\frac{1}{x}$. Why does this not contradict the theorem?

11. If a resistor of r ohms is connected across a battery of V volts with internal resistance R ohms, then the power in watts in the external resistor is $P(r)=\frac{V^2r}{(r+R)^2}$. If V and R are constant by r varies, what is the maximum value of the power?

12. Consider the equation $\sin 3x=1-x^3$
    a. Use the intermediate value theorem to prove that this equation has a solution in $\left[0,\frac{\pi}{3}\right]$
    b. Use Newton's method to find the next estimate to the solution starting from $x_1=\frac{\pi}{6}$.

13. Find the antiderivative of $f(x)=\frac{8}{x^2}$ which has $y=x$ as a tangent line.