Math 1A – Chapter 1 Test – Spring '08 **Name________________________**

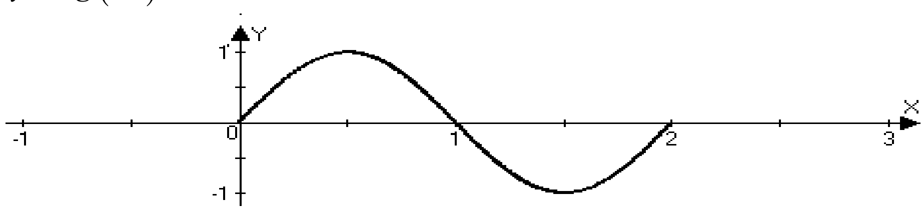
Write all responses on separate paper. Show your work for credit. Do not use a calculator.

1. Find an equation for the parabola with its vertex at (2,5) and passing through (0,0).

2. Consider the function $f(x)=x^2$. Find the slope of the line from $(0,0)$ to $(a, f(a))$ in terms of a.

3. Given that $f(x)=\frac{1}{x}$, simplify a formula for the function $g(x)=1+2f(x+3)$ and sketch a graph for $y = g(x)$ showing asymptotes and intercepts.

4. The graph of a function $y=g(x)$ is given below. Use it to graph each of the following:
   a. $y=2+g(x+1)$
   b. $y=3g(2x)$



5. Consider the logarithmic function $f(x)=\log_3(2x+3)$
   a. Find the x- and y-intercepts for this function.
   b. Find a formula for the inverse function.
   c. Graph the function and its inverse together, showing the symmetry through the line $y = x$.

6. Solve the given equation in exact form and then approximate to 4 digits, if appropriate.
   a. $\log_2(4x+4)+\log_2(x+1)=10$
   b. $10-e^{1-x^2}=8$

7. Suppose a point in the fourth quadrant ($x > 0$ and $y < 0$) of the unit circle has x coordinate $\frac{3}{5}$.
   a. Find the y-coordinate of this point and draw the unit circle in the x-y plane showing its position.
   b. Suppose θ is a polar angle that terminates at this point. What are the coordinates of the terminal point for $\theta-\pi$?

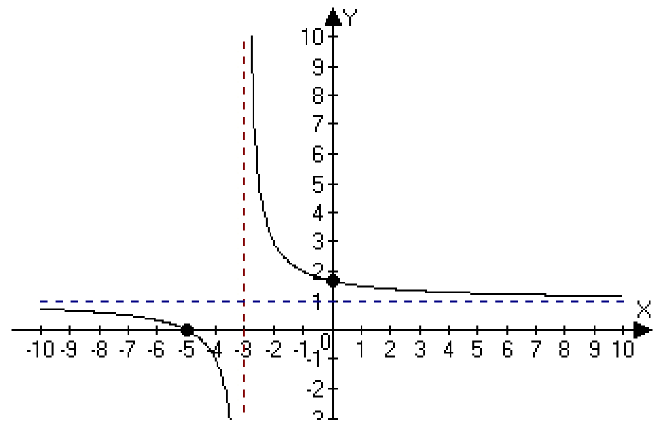
8. Consider the function $f(x)=1+2\sin\left(x+\frac{\pi}{3}\right)$ Find the (a) amplitude, (b) period and (c) phase shift for this sine function and sketch a graph showing at least one wave form and its intercepts.

9. Find a parameterization of the circle described by $x^2+y^2=4$ using trigonometric functions.

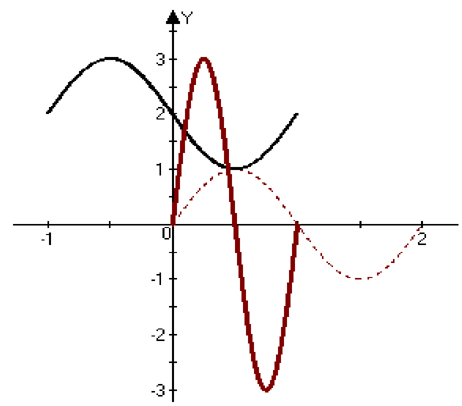
10. Find a Cartesian (rectangular) equation for the curve described by parametric equations
    $x=2\cos t$
    $y=2+3\sin t$

Math 1A – Chapter 1 Test Solutions – Spring '08

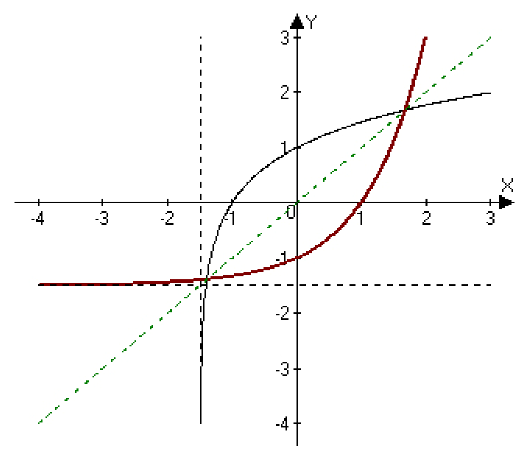
1. Find an equation for the parabola with its vertex at (2,5) and passing through (0,0).
   SOLN: Plug the coordinates of the vertex into the vertex form $y = a(x-h)^2 + k$ and you have $y = a(x-2)^2 + 5$. Now if it goes through (0,0) then $0 = a(0-2)^2 + 5 \Leftrightarrow a = -\frac{5}{4}$ so the equation is $y = -\frac{5}{4}(x-2)^2 + 5 \Leftrightarrow y = -1.25x^2 + 5x$
2. Consider the function $f(x) = x^2$. Find the slope of the line from $(0,0)$ to $(a, f(a))$ in terms of a.
   SOLN: $\frac{\Delta y}{\Delta x} = \frac{f(a) - f(0)}{a - 0} = \frac{a^2}{a} = a$
3. Given that $f(x) = \frac{1}{x}$, simplify a formula for the function $g(x) = 1 + 2f(x+3)$ and sketch a graph for $y = g(x)$ showing asymptotes and intercepts.
   SOLN:
   $g(x) = 1 + 2f(x+3) = 1 + \frac{2}{x+3} = \frac{x+5}{x+3}$ has horizontal asymptote along $y = 1$ and vertical asymptote along $x = -3$. The intercepts are (0,5/3) and (–5,0).



4. The graph of a function $y = g(x)$ is given below. Use it to graph each of the following:
   SOLN: (The original is dotted at right.)
   a. $y = 2 + g(x+1)$
      This is a shift 2 up and 1 to the left.
      Note that the shape is congruent to the original (The is thicker plot starting (-1,2).
   b. $y = 3g(2x)$
      This is a horizontal compression by factor 2 and a vertical stretch by factor 3 – the thickest plot shown at right.



5. Consider the logarithmic function $f(x) = \log_3(2x+3)$
   a. Find the x- and y-intercepts for this function.
      SOLN: $f(0) = \log_3(3) = 1$ and $f(-1) = \log_3(1) = 0$ so the intercepts are at (0,1) and (–1 ,0).
   b. Find a formula for the inverse function. SOLN:
      $y = \log_3(2x+3) \Leftrightarrow 2x + 3 = 3^y \Leftrightarrow$
      $x = \frac{3^y - 3}{2} = 0.5(3^y) - 1.5 \Rightarrow \boxed{f^{-1}(x) = 0.5(3^x) - 1.5}$
   c. Graph the function and its inverse together, showing the symmetry through the line $y = x$ SOLN: at right:



6. Solve the given equation in exact form and then approximate to 4 digits, if appropriate.

   a. $\log_2(4x+4)+\log_2(x+1)=10$ SOLN:

   $\log_2(4x+4)+\log_2(x+1)=\log_2(4x+4)(x+1)=10 \Leftrightarrow 4(x+1)(x+1)=2^{10} \Leftrightarrow (x+1)^2=2^8 \Leftrightarrow x=-1\pm 2^4$

   But $x>-1$ so $\boxed{x=-1+16=15}$ is the only real solution.

   b. $10-e^{1-x^2}=8$ SOLN:

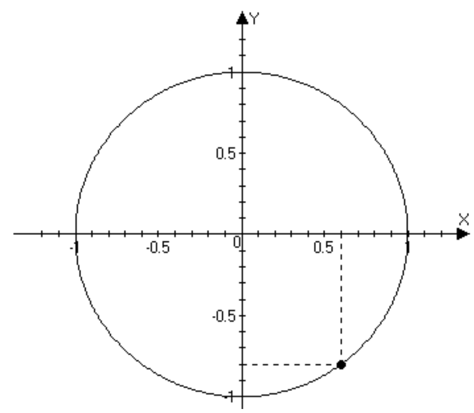
   $10-e^{1-x^2}=8 \Leftrightarrow e^{1-x^2}=2 \Leftrightarrow 1-x^2=\ln 2 \Leftrightarrow x^2=1-\ln 2 \Leftrightarrow x=\pm\sqrt{1-\ln 2}$

7. Suppose a point in the fourth quadrant ($x>0$ and $y<0$) of the unit circle has x coordinate $\frac{3}{5}$.

   a. Find the y-coordinate of this point and draw the unit circle in the x-y plane showing its position.

   SOLN: $y=-\sqrt{1-\left(\frac{3}{5}\right)^2}=-\sqrt{1-\frac{9}{25}}=-\sqrt{\frac{16}{25}}=-\frac{4}{5}$

   So the point is at (0.6, – 0.8). The plot is shown at right.



   b. Suppose θ is a polar angle that terminates at this point. What are the coordinates of the terminal point for $\theta-\pi$?

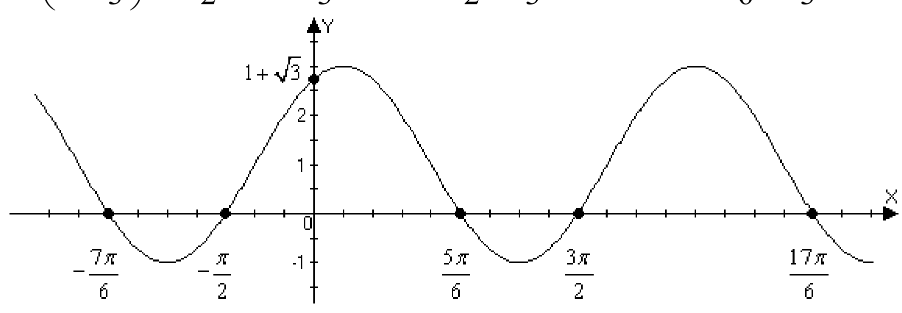
   SOLN: This would be the point reflected through the origin (rotated 180°) whose coordinates are (–0.6, 0.8).

8. Consider the function $f(x)=1+2\sin\left(x+\frac{\pi}{3}\right)$ Find the (a) amplitude, (b) period and (c) phase shift for this sine function and sketch a graph showing at least one wave form and its intercepts.

   SOLN: The amplitude = 2, the period is 2π and the phase shift is $-\frac{\pi}{3}$. The y-intercept is at

   $f(0)=1+2\sin\left(\frac{\pi}{3}\right)=1+\sqrt{3}$ and the x-intercepts are where

   $\sin\left(x+\frac{\pi}{3}\right)=-\frac{1}{2} \Leftrightarrow x+\frac{\pi}{3}+2\pi k=\frac{3\pi}{2}\pm\frac{\pi}{3}+2\pi k \Leftrightarrow x=\frac{7\pi}{6}\pm\frac{\pi}{3}+2\pi k$



9. Find a parameterization of the circle described by $x^2+y^2=4$ using trigonometric functions.

   SOLN: $x=2\cos t;\ y=2\sin t$

10. Find a Cartesian (rectangular) equation for the curve described by parametric equations $x=2\cos t;\ \ y=2+3\sin t$

    SOLN: $\left(\frac{x}{2}\right)^2+\left(\frac{y-2}{3}\right)^2=1$