Math 1A – Chapter 3 Test – Typical Problems Set

1. Use the definition of the derivative to compute each of the following limits. Follow the form of this example: $\lim_{x\to 2}\frac{x^{10}-1024}{x-2}=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=f'(2)=10(2)^9=5120$

   a. $\lim_{x\to 0}\frac{(x^6+x+1)-1}{x}$ b. $\lim_{x\to 1}\frac{2^{3x}-8}{x-1}$ c. $\lim_{x\to 1/3}\frac{2\cos(\pi x)-1}{x-\frac{1}{3}}$

2. Find equations of the lines tangent to the curve $y=\frac{x}{1+x^2}$ which are parallel to the line $y-0.48x=0$. Sketch a graph illustrating these tangencies.

3. Newton's law of gravitation says that the magnitude *F* of the force exerted by a body of mass *m* on a body of mass *M* is

$$F=\frac{GmM}{r^2}$$

   a. Assume the bodies are in motion. Find *dF*/*dr* and write a sentence of two explaining what that means.

   b. Suppose that planet Xorkon attracts an object with a force that decreases at a rate of 3 N/km when *r* = 10,000 km. How fast does the force change when *r* = 5000 km?

4. Use the *definition* of the derivative to simplify $\frac{d}{d\theta}\cos(2\theta)$.

5. For what values of *x* does the graph of $f(x)=x^3+6x^2+x+4$ have a horizontal tangent?

6. A curve *C* is defined by the parametric equations $\begin{matrix} x=\sin(2t) \\ y=2\cos(t) \end{matrix}$.

   a. Show that *C* has two tangent lines at the origin: $(x,y)=(0,0)$ and find their equations.
   b. Find the points where the tangent line in the *x*-*y* plane is vertical.

7. Find an equation for the line tangent to $x^2y+xy^2=2xy$ at (1,1).

8. Let $y=\sqrt[3]{x}$.
   a. Find the differential *dy*.
   b. Evaluate *dy* and Δy if $x = 8$ and $dx = \Delta x = 0.1$
   c. Estimate $\sqrt[3]{8.1}$ using the line tangent to $y=\sqrt[3]{x}$ at (8,2). What is the relative error in your estimation?

9. Find the derivative of the function $f(x)=(\cos x)^x$ by first differentiating $\ln y$.

10. Consider $f(r)=2\sqrt{r}-3\sqrt[3]{r}$.
    a. Simplify formulas for the first and second derivatives of *f*(*r*).

b. Use calculus to find the coordinates of the inflection point for $f(r)$.

11. Find equations for the tangent lines to $y=\frac{x+1}{x-1}$ that are parallel to the line $x+2y=17$.

12. If $h(9) = 3$ and $h'(9) = -7$, find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=9}$

13. In a fish farm, a population of the fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation
$$\frac{dP}{dt}=r_0\left(1-\frac{P(t)}{P_{\max}}\right)P(t)-H\cdot P(t)$$
where r_0 is the birth rate of the fish, $P_{\max}$ is the maximum population the pond can sustain and H is proportion of fish harvested in a year. If the pond can sustain a maximum population of 5,000 fish, the birth rate is 4% and the harvesting rate is 2%, what
(non-zero) population level(s) will not change, according to the model?

14. The gas law for an ideal gas at absolute temperature T (in kelvins), pressure P (in atmospheres), and volume V (in liters) is $PV = nRT$, where n is the number of moles of the gas and $R = 0.0821$ is the gas constant. Suppose that, at a certain instant, $P = 8$ atm and is increasing at a rate of 0.14 atm/min and $V = 11$L and is decreasing at a rate of 0.17 L/min. Find the rate of change of T with respect to time at that instant if $n = 10$ moles. Round your answer to four decimal places. $\frac{dT}{dt}\approx$? K/min

15. Use the definition of the derivative to compute $\frac{d}{dx}\sec(x)$.

16. Show that the curve described by the parametric equations $\begin{aligned}x(t)&=1-\cos(\pi t)\\ y(t)&=\sin 3\pi t\end{aligned}$
has two tangent lines where $t = 1/3$ and find their equations. Illustrate these in a graph.

17. If $\sqrt{3}\sin x\cos y=1$, find a formula for $\frac{dy}{dx}$ using implicit differentiation.

18. Find $\frac{d^{63}}{dx^{63}}(x\sin x)$ by finding the first few derivatives and observing the pattern that occurs.

19. Use logarithmic differentiation to to find the derivative of $y=(\sec x)^x$

20. Verify the given linearization $\ln|x+1|\approx x$ near $x = 0$. What interval for x for is the linear approximation accurate to within 0.1?

21. Find an equation for the line tangent to $y=\tan^{-1}(\sin x)$ where $x = 0$.

22. Find an equation of the tangent line to the parametric curve $x=e^{\sqrt{t}},\quad y=-t^6+t$ at the point corresponding to $t = 1$.

23. Find y'' if $x^8+y^8=1$.

24. Find an equation for the line tangent to the curve $y^3 + 3x^2 y + x^3 = 15$ at (1,2).

25. Use implicit differentiation to find an equation of the tangent line to the curve

at the point (4, 2). Give your answer in the form

26. At what point on the curve $y = 12x^{2/3} - 4x$ is the tangent line horizontal?

27. Derive a simple formula for the derivative — .

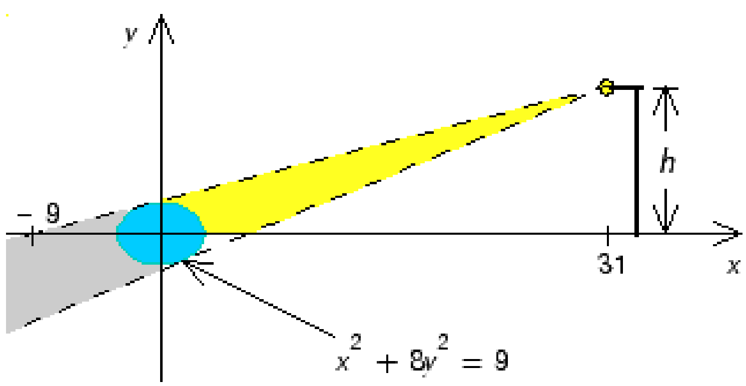
28. Find the points on the hyperbola $x^2 - 2y^2 = 1$ where the tangent line has slope = 2.

29. Let —

a. Find
b. Find

30. Find a parabola that passes through (1,10) and whose tangent lines at $x = -2$ and $x = 1$ are –5 and 7, respectively.

31. The figure shows a lamp located 31 units to the right of the y-axis and a shadow created by the elliptical region .
If the point is on the edge of the shadow, how far above the x-axis is the lamp located?



32. Find an equation for the line tangent to the curve described by the parametric equations

$$x = \sin(3t) - \cos t$$
$$y = \cos(3t) - \sin t$$

where $t = \frac{\pi}{6}$.

33. Find if .

34. Use a linear approximation (or differentials) to estimate ——— .

35. A particle moves on a horizontal line so that its coordinate at time t is $x = e^{-t/4}\cos(2t)$.

a. Find the velocity and acceleration functions.
b. Find the distance the particle travels in the time $0 \le t \le \pi$.

c. When is the particle speeding up? When is it slowing down?

36. Consider $f\left(x\right)=\sqrt[3]{999+x^3}$

a. Find the linearization of at $x = 9$ and use it to approximate $\sqrt[3]{1999}$.
b. What is the % error in your approximation?

37. Find the coordinates of the points where the line through (0,3) is tangent to the unit circle, $x^2+y^2=1$.