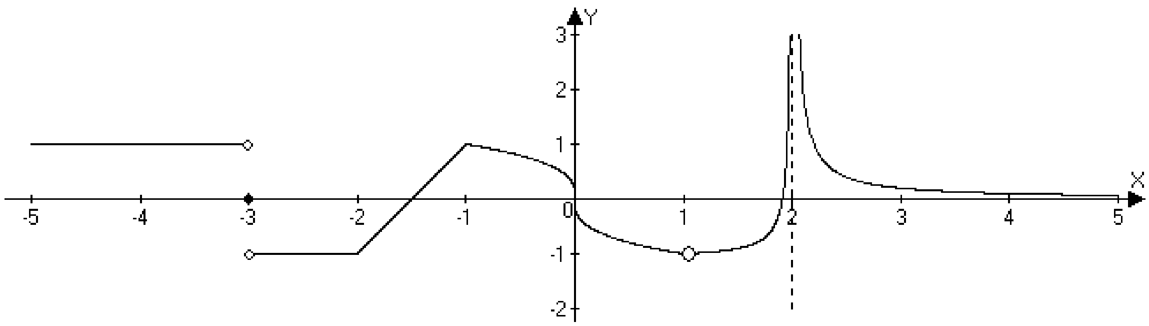
Math 1A – Final Exam – Fall ’04

1. The graph below shows $y = f(x)$.
   a. State, with reasons, the numbers a at which $\lim_{x \to a} f(x)$ does not exist.
   b. State where the function is discontinuous and classify the discontinuity as removable, jump or asymptote.
   c. State, with reasons, the numbers at which the function is not differentiable.

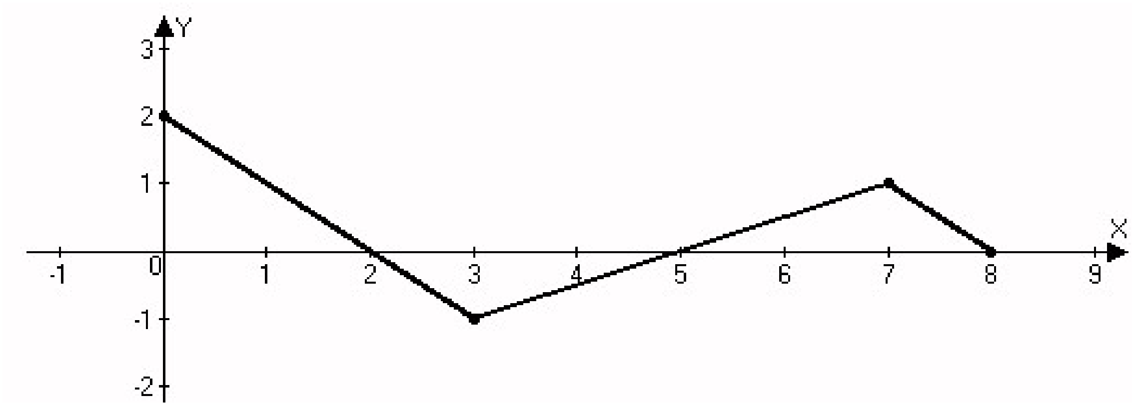


2. Suppose $f(x)$ is a function such that for all $x > 0$, $\frac{2}{\pi}\arctan x < f(x) < \frac{x+1}{x-1}$.

   What is $\lim_{x \to \infty} f(x)$? Why?

3. Sketch a graph for a function that meets the given conditions:
   $f(0) = 0, \quad f'(-4) = f'(0) = f'(4) = 0$
   $f'(x) < 0$ on $(-\infty, -4) \cup (-4, 0) \cup (4, \infty)$
   $f''(x) > 0$ on $(-\infty, -4) \cup (-2, 2) \cup (6, \infty)$

4. The graph of $f'(x)$ is shown below.
   a. Sketch a graph for $f''(x)$.
   b. Sketch a possible graph for $f(x)$.



5. Find a linear approximation for the function $f(x) = \sin\left(x + \frac{\pi}{3}\right)$ at $a = 0$ and use it to approximate $f(-0.14)$.

6. Find the slope of the line tangent to the curve $x^2 \cos y + \sin 2y = xy$ at $(x,y) = \left(0, \frac{\pi}{2}\right)$.

7. Find an equation for the line tangent to $y = e^{\sin 3x}$ at $x = \frac{\pi}{3}$.

8. At what point on the curve $y = \left[\ln(x+4)\right]^2$ is the tangent line horizontal?

9. Find the local and global extreme values of the function $y = x^3 e^{-x^2}$ on the interval [-1,2].

10. Find the point on the hyperbola $xy = 16$ that is closest to the point (4,0).

11. Evaluate the limit: $\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$.

12. The velocity of a wave of length L in deep water is $v = k\sqrt{\frac{L}{C} + \frac{C}{L}}$ where k and C are known positive constants. What is the length of the wave that gives the minimum velocity?

13. What is the maximum slope of a line connecting the origin (0,0) with a point on the parabola $y = 1 - (x-2)^2$?

14. Show that the y-coordinate of the point (x,y) on the curve described by $\frac{x^2+1}{\ln(y^2+4)} = 1$ that is closest to the point (0,2) can be found by solving $2y^3 - 4y^2 + 10y - 16 = 0$. Use Newton's method to solve the equation. *Hint*: Minimizing the distance D is equivalent to minimizing the square of the distance, D^2. Also, the curve has origin symmetry, so you can just work with the distance in the first quadrant.

15. A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder with a hemispherical top. What radius and height will require the least amount of metal?