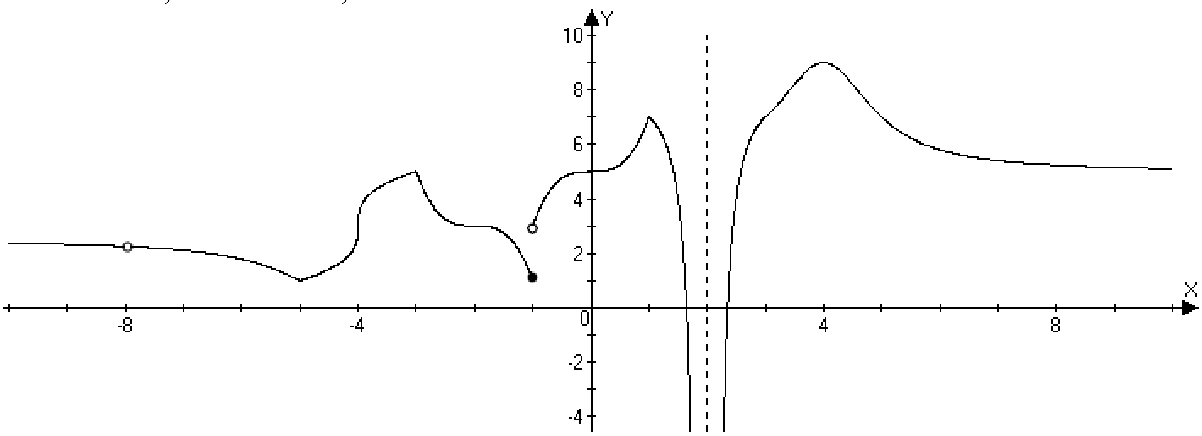
Math 1A – Final Exam – spring '07 Name___________________

Show your work for credit. Write all responses on separate paper. Do not abuse a calculator.

1. The graph below shows $y = f(x)$.
   a. State, with reasons, the numbers a at which $\lim_{x\to a} f(x)$ does not exist.
   b. State where the function is discontinuous and classify the discontinuity as removable, jump or asymptote.
   c. State, with reasons, the numbers at which the function is not differentiable.



2. Compute the limit, if it exists.
   a. $\lim_{x\to 2} \dfrac{x-2}{x^2-3x+2}$
   b. $\lim_{h\to 0} \dfrac{\sqrt{h+1}-1}{h}$
   c. $\lim_{x\to \infty} \dfrac{2x^4-x^2+4x}{16-x^4}$
   d. $\lim_{\theta\to 0} \dfrac{\sin 5\theta}{2\theta}$

3. Use the definition of the derivative to derive one of the formula below (your choice)

   $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$ or $\dfrac{d}{dx}(\sin x) = \cos x$

4. Use the various shortcut rules (product rule, chain rule, etc.) to simplify the derivative:
   a. $f(t) = \dfrac{3t-5}{t^2+7t-11}$
   b. $g(x) = x^5 \tan(2x)$
   c. $h(u) = \sin(\sin(\sin u))$

5. Sketch a graph for a function defined on [-4,4] that meets the given conditions:

   $f(0)=1,\quad f'(-2)=f'(1)=f'(3)=0,\ f'(0)$ is undefined

   $f'(x)<0$ only on $(-2,0)\cup(1,3)$

   $f''(x)<0$ only on $(-4,0)\cup(0,2)$

6. Find a linear approximation for the function $R = \dfrac{100r}{100+r}$ near $a = 50$ and use it to approximate R when $r = 53$.

7. Find an equation for the line tangent to $y = e^{\cos 2x}$ at $x = \dfrac{\pi}{4}$.

8. Under what condition(s) will the points on a tangent line lie beneath the curve?

9. Find the local and global extreme values of the function $y = x^2 \ln x^2$ on the interval [-2,2].

10. Find the point on the parabola $x = 16 - y^2$ that is closest to the point (16, 3).

11. If $y = e^{xy}$, find an expression for $\dfrac{dy}{dx}$ and write a linearization for y near (0,1).

12. Find an equation for the line tangent to $x^3 + x^2 y + 2y^3 = 2$ at $(2,-1)$.

13. Show that the y-coordinate of the point (x,y) on the curve described by $\dfrac{x^2+1}{\ln\left(y^2+4\right)} = 1$ that is closest to the point (0,2) can be found by solving $2y^3 - 4y^2 + 10y - 16 = 0$. Use Newton's method to solve the equation. *Hint*: Minimizing the distance D is equivalent to minimizing the square of the distance, D^2. Also, the curve has origin symmetry, so you can just work with the distance in the first quadrant.

14. A box with a square base with no top has a surface area of 108 square feet. Find the dimensions that will maximize the volume.

15. Use Newton's method to approximate a solution to $x^2 = \cos 2x$ to 8 digits. Simplify the iteration formula for this case, use an initial value of $x = 1$ and show what the iterates are.