Some 3.10 (linearization) problems

Find the linear approximation of the function $f(x) = \sqrt{1-x}$ near $x = 0$, and use it

to approximate $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate in a graph.

ANS: $f'(x) = \frac{d}{dx}(1-x)^{\frac{1}{2}} = \frac{du}{dx} \cdot \frac{d}{du} u^{\frac{1}{2}}$ where $u = 1 - x$.
So $f'(x) = -1 \cdot \frac{1}{2}(1-x)^{-\frac{1}{2}} = -\frac{1}{2\sqrt{1-x}}$

$$L(x) = f(0) + f'(0)(x-0) = 1 - \frac{1}{2}x$$

$$\sqrt{0.9} = \sqrt{1-0.1} \approx 1 - \frac{1}{2}(0.1) = 0.95$$

The calculator approx. is $\sqrt{0.9} \approx 0.948683$

For $\sqrt{0.99} \approx 1 - \frac{1}{2}(0.01) = 0.995$ is even better:

$$\sqrt{0.99} \approx 0.994987$$

3.10 #14) Find the differential for $f(\theta) = \ln(sin(\theta))$

ANS: $dy = f'(\theta)d\theta = \frac{d}{d\theta}\ln(sin(\theta)) = \frac{du}{d\theta} \cdot \frac{d}{du}\ln u = cos\theta \cdot \frac{1}{u} = \frac{cos\theta}{sin\theta}$

$$\frac{dy}{d\theta} = \frac{cos\theta}{sin\theta} \Leftrightarrow dy = \frac{cos\theta}{sin\theta}d\theta$$

Recall $\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{f'(x)}{\left(f(x)\right)^2}$

(b) $f(x) = \frac{e^x}{1-e^x} = -\frac{1}{1-e^{-x}} \Rightarrow dy = \frac{e^{-x}}{(1-e^{-x})^2}dx$

3.10 #28) Use a linear approximation (or differentials) to estimate $\cos 29°$.

$$29° \frac{\pi}{180°} = \frac{29\pi}{180}$$

Near $x = \frac{\pi}{6}$

$$f(x) = \cos(x) \approx f\left(\frac{\pi}{6}\right) + f'\left(\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right) = y_0 + dy$$

$$f\left(\frac{29\pi}{180}\right) \approx \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \left(-\frac{\pi}{180}\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{360} \approx 0.874752$$

The calculator approximation if $\cos 29° \approx 0.874620$