Math 1A–Spring '17 **Name (Print):** ____________________
Chapter 4 Test

Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (10 points) Find the local and absolute extreme values of the function $f(x) = x + 2\sin(2x)$ on the interval $[0, \pi]$.

2. (12 points) Evaluate the limit.

   (a) $\lim_{x\to 0^+} \dfrac{\tan(x^2)}{x\sin(x)}$

   (b) $\lim_{x\to 0^+} (\cos(x))^{1/x}$

3. (10 points) Sketch the graph of a function that satisfies the given conditions:
   $f(0) = 0$,
   f is odd and continuous,
   $f'(x) > 0$ on $(0, 2)$,
   $f'(x) < 0$ on $(2, \infty)$,
   $f''(x) < 0$ on $(0, 3)$
   $f''(x) > 0$ on $(3, \infty)$ and
   $\lim_{x\to\infty} f(x) = 0$.

4. (16 points) Suppose $f(x) = xe^{-x^2}$

   (a) What is the domain of f?

   (b) What intercept(s) does f have?

   (c) Describe the symmetry of f.

   (d) What asymptote(s) does f have?

   (e) Simplify $f'(x)$ and use it to find critical numbers and intervals of increase/decrease.

   (f) Simplify $f''(x)$ and use that to find inflection points of f.

   (g) Sketch a graph illustrating all these components of f.

5. (12 points) Consider $g(x) = x^3 - x$ on $[-2, 2]$

   (a) Explain why this function satisfies the premis of the Mean Value Theorem on that interval.

   (b) Find all values of $c \in (-2, 2)$ that safisfy the conclusion of the Mean Value Theorem.

6. (12 points) Find the minimum value of $\ln(x^2 + y)$ so that $x + y = 5$.

7. (12 points) Find the volume of the largest cone that can be inscribed in a sphere of radius R.

8. (16 points) To find the cube root of a number, A, we want to find a zero of the polynomial $p(x) = x^3 - A$

   (a) Show that Newton method for finding the cube root of A is equivalent to iterating

   $$x_{n+1} = \frac{2x_n + \dfrac{A}{x_n^2}}{3}$$

   (b) Starting with $x_1 = 1$, find x_2 and x_3 in the Newton's method search for $\sqrt[3]{3}$