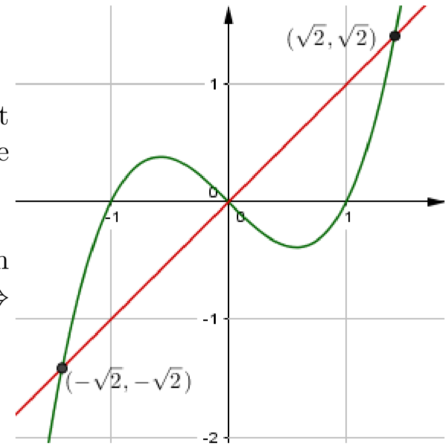
1. Differentiate the function.
   (a) $f(x) = \dfrac{x^3+2x-1}{\sqrt{x}}$
   ANS: $\dfrac{d}{dx}f(x) = \dfrac{d}{dx}\left(\dfrac{x^3}{\sqrt{x}} + \dfrac{2x}{\sqrt{x}} - \dfrac{1}{\sqrt{x}}\right) = \dfrac{d}{dx}(x^{5/2} + 2\sqrt{x} - x^{-1/2}) = \frac{5}{2}x^{3/2} + \dfrac{1}{\sqrt{x}} + \frac{1}{2}x^{-3/2} = \dfrac{5x^3+2x+1}{2x^{3/2}}$
   (b) $R(k) = e^{2k} - k^e$
   ANS: $2e^{2k} - ek^{e-1}$

2. At what other point(s) does the normal line to the function $f(x) = x^3 - x$ at the point $(0,0)$ intersect the function? Illustrate with a sketch. *Recall:* The normal line is the line perpendicular to the tangent line.

   ANS: $f'(x) - 3x^2 - 1$ so $f'(0) = -1$ so the normal line passes through $(0,0)$ with slope $m = 1$. This is the line $y = x$ which intersects $f(x)$ where $x = x^3 - x \Leftrightarrow x(x^2-2) = 0$. The coordinates of the other two points are thus $(\pm\sqrt{2}, \pm\sqrt{2})$.



3. Use the **definition** of the derivative (that is, $f'(x) \equiv \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$) to compute $\dfrac{d}{dx}\cos(x)$
   ANS: $\dfrac{d}{dx}\cos(x) = \lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos(x)}{h} = \lim\limits_{h\to 0}\dfrac{\cos(x)\cos(h) - \sin(h)\sin(x) - \cos(x)}{h}$

   $$= \lim_{h\to 0}\cos(x)\frac{\cos(h)-1}{h} - \sin(x)\frac{\sin(h)}{h} = \cos(x)\lim_{h\to 0}\underbrace{\frac{\cos(h)-1}{h}}_{\to 0} - \sin(x)\lim_{h\to 0}\underbrace{\frac{\sin(h)}{h}}_{\to 1} = -\sin(x)$$

4. Suppose that $g(\pi) = 2$ and $g'(\pi) = -3$. Evaluate $\dfrac{d}{dx}\left(\dfrac{x}{g(x)}\right)\Big|_{x=\pi}$.
   ANS: $\dfrac{d}{dx}\left(\dfrac{x}{g(x)}\right)\Big|_{x=\pi} = \dfrac{g(x) - xg'(x)}{(g(x))^2}\Big|_{x=\pi} = \dfrac{g(\pi) - \pi\cdot g'(\pi)}{(g(\pi))^2} = \dfrac{2-\pi\cdot(-3)}{4} = \dfrac{3\pi+2}{4}$

5. Find the equation(s) of the tangent line(s) for the curve $y = \arctan(4x)$ which are parallel to $2x - y = 17$
   ANS: We want the slope of the tangent line, $y' = \dfrac{4}{1+16x^2}$ to match the slope of the given line, $m = 2$, so solve $\dfrac{4}{1+16x^2} = 2 \Leftrightarrow x = \pm\frac{1}{4}$. Thus the equations for the tangent lines are $\boxed{y \pm \dfrac{\pi}{4} = 2(x \pm \frac{1}{4})}$

6. A particle moves along a straight line with displacement $s(t)$, velocity $v(t)$, and acceleration $a(t)$. Show that

   $$a(t) = v(t)\frac{dv}{ds}$$

   Explain the difference between the meanings of the derivatives dv/dt and dv/ds.
   ANS: The acceleration is the time rate of change of the velocity: $a(t) = \dfrac{dv}{dt}$ By the chain rule, this is $a(t) = \dfrac{dv}{ds}\dfrac{ds}{dt}$, but $\dfrac{ds}{dt} = v(t)$, so there you are! Note that $\dfrac{dv}{ds}$ is the instantaneous rate of change of the velocity per change in displacement.

7. Consider the equation $x\cos y + y\cos x = \pi$.

(a) Differentiate implicitly to find an expression for $\dfrac{dy}{dx}$ in terms of x and y.

ANS:

$$\begin{aligned}\cos y - x\sin y\frac{dy}{dx} + \frac{dy}{dx}\cos x - y\sin x &= 0\\ (\cos x - x\sin y)\frac{dy}{dx} &= y\sin x - \cos y\\ \frac{dy}{dx} &= \frac{y\sin x - \cos y}{\cos x - x\sin y}\end{aligned}$$

(b) Find an equation for the line tangent to the curve described by the equation at $(\pi, 0)$.

Plug into the point-slope template: $y - y_0 = m(x - x_0) \Rightarrow y = 1\cdot(x-\pi) \Leftrightarrow y = x - \pi$

As a bonus, we use Geogebra to examine the implicit curve and its tangent at that point: