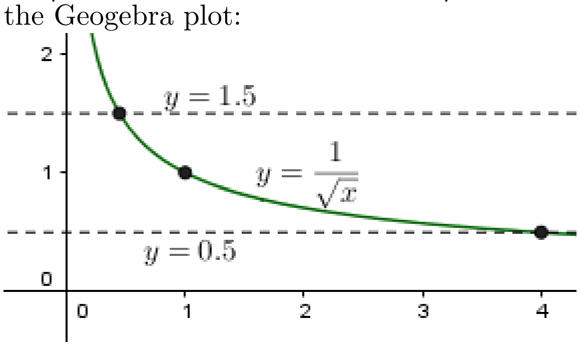
Math 1A - Spring 2017 - Chapter 2 Review Quiz

1. Let $f(x) - \dfrac{1}{\sqrt{x}}$. Find the largest δ so that $\|x - 1\| < \delta$ will guarantee $\|f(x) - 1\| < \frac{1}{2}$.
SOLN:

$$\begin{aligned} |f(x) - 1| &< \frac{1}{2} \\ \left|\frac{1}{\sqrt{x}} - 1\right| &< \frac{1}{2} \end{aligned}$$

The boundary of the inequality is where you have equality, which can happen here if either $\dfrac{1}{\sqrt{x}} - 1 = \frac{1}{2}$ or $\dfrac{1}{\sqrt{x}} - 1 = -\frac{1}{2}$. That is, either $\dfrac{1}{\sqrt{x}} = \frac{3}{2}$ or $\dfrac{1}{\sqrt{x}} = \frac{1}{2}$ which is true if $x = \frac{4}{9}$ or $x = 4$. This is seen in the Geogebra plot:



The closer point is

2. Use the definition of the derivative to find $f'(x)$.
SOLN:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}}}{h} \\ &= \lim_{h\to 0} \frac{\dfrac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\ &= \lim_{h\to 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\ &= \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = \frac{-1}{2x\sqrt{x}} \end{aligned}$$

3. Find an equation for the line tangent to the function at $(4, \frac{1}{2})$.
SOLN: Use the form $y - f(4) = f'(4)(x - 4) \Leftrightarrow y - \frac{1}{2} = -\dfrac{1}{16}(x - 4) \Leftrightarrow \boxed{y = -\dfrac{1}{16}x + \dfrac{3}{4}}$