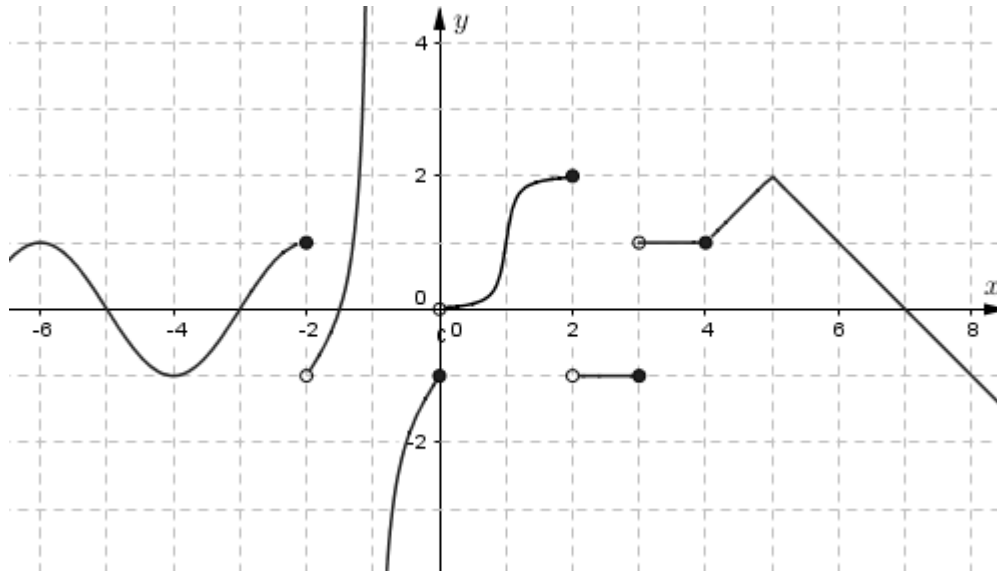


Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (18 points) Consider the graph of the function $y = f(x)$ shown below:



(a) Find each of the following, or explain why it does not exist.

i. $f(-2)$

iii. $\lim_{x \rightarrow 0^-} f(x)$

v. $\lim_{x \rightarrow 2^+} f'(x)$

ii. $\lim_{x \rightarrow -2^+} f(x)$

iv. $\lim_{x \rightarrow 2} f(x)$

vi. $\lim_{x \rightarrow 3} f'(x)$

(b) Is $f'(x)$ discontinuous where $x = 5$? Justify your answer using the definition of continuity.

(c) Assume that f has a vertical asymptote along $x = -1$ as suggested by the graph. Is $\lim_{x \rightarrow -1} f'(x) = \infty$?
Why or why not?

2. (12 points) Suppose

$$f(x) = \begin{cases} x + 8 & \text{if } x < 2 \\ x^2 + 3 & \text{if } x > 2 \end{cases}$$

$y = g(x)$:

and that $g(x)$ is defined by the graph shown at right.

(a) What is $\lim_{x \rightarrow 2^-} f(x)$

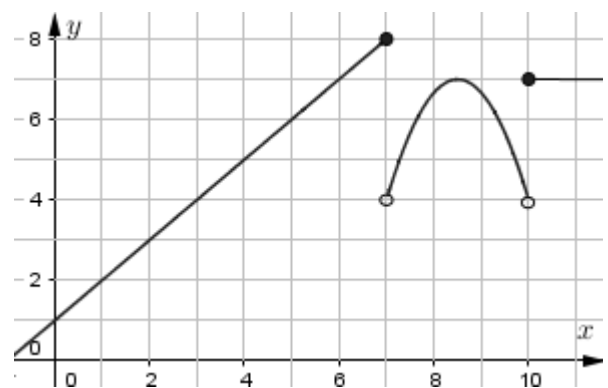
(b) What is $\lim_{x \rightarrow 2^+} f(x)$

(c) What is $\lim_{x \rightarrow 2} g(f(x))$

(d) Can you apply the theorem that concludes that

$$\lim_{x \rightarrow 2} g(f(x)) = g\left(\lim_{x \rightarrow 2} f(x)\right)$$

Why or why not?



3. (8 points) Let $P(t)$ = the inches of precipitation at Fantasy Springs on day t where t = the number of days since 1/1/2017. The table at right shows the value of this function over a 5 day period.

Date	$P(t)$ (in.)
1/1/2017	0.9
1/2/2017	1.1
1/3/2017	1.2
1/4/2017	0.9
1/5/2017	1.6

- (a) Use the table to find the average rate of change in precipitation between 1/1/2017 and 1/5/2017. Be sure to specify the units of measure for this rate of change.
- (b) What is your best approximation, based on this table, for rate of change on January 3?
4. (18 points) Let $f(x) = \sqrt{x}$
- (a) Find the largest δ so that $|x - 4| < \delta$ guarantees that $|f(x) - 2| < \frac{1}{2}$
- (b) Use the definition of the derivative as a limit to find the derivative function $f'(x)$.
- (c) Find an equation for the line tangent to $y = f(x)$ at $x = 4$.
5. (6 points) Find the smallest value of N so that if $x > N$ then $\frac{\pi}{2} - \arctan(x) < \frac{\pi}{6}$

6. (8 points) Let

$$f(x) = \begin{cases} ax + b & : x < 1 \\ 3 & : x = 1 \\ bx - a & : x > 1 \end{cases}$$

Find values of a and b so that f is a continuous function.

7. (10 points) Use the intermediate value theorem to show that the equation $x^3 + x = 1$ has a real solution. First state the Intermediate Value Theorem, then show precisely how the premise is satisfied and what conclusion follows.

8. (10 points) Find a function f and a number a such that $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h} = f'(a)$.

9. (10 points) The figure shows the graphs of f , f' , and f'' . Identify each curve, and explain your choices.

