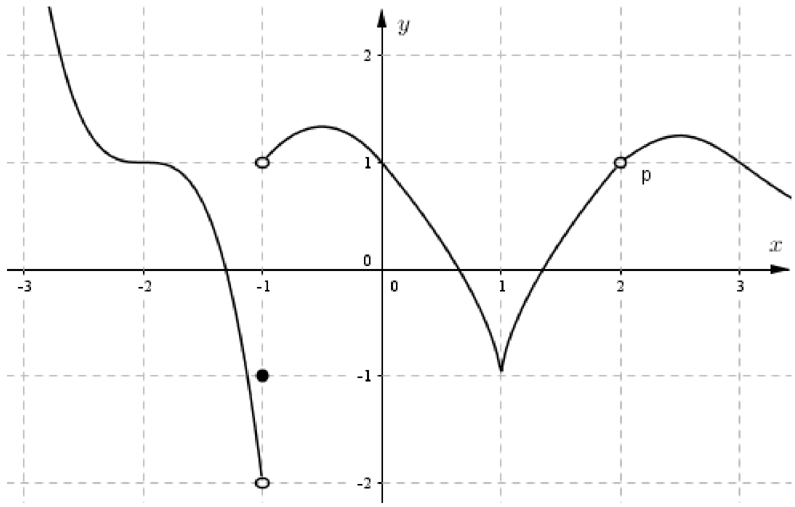
Math 1A **Name (Print):** ____________________

Final Exam, Spring 2016

Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (12 points) Consider the function defined by its schematic graph below.



   (a) Find each limit or write "DNE" if the limit does not exist.

   i $\lim_{x\to -1^-} f(x)$ ii $\lim_{x\to -1^+} f(x)$ iii $\lim_{x\to 2} f(x)$

   (b) Find all the discontinuities and classify each as either a removable discontinuity, a jump discontinuity or a vertical asymptote.

   (c) Approximate x for all points where $f'(x) = 0$.

   (d) Approximate x for all points where $f'(x) = -\frac{1}{2}$ (approximate to the nearest tenth.)

2. (12 points) Use the **definition** of the derivative (that is, $f'(x) \equiv \lim_{a\to x} \frac{f(x)-f(a)}{x-a}$) to show that $\frac{d}{dx}\left(\frac{2}{x}\right) = -\frac{2}{x^2}$.

3. (12 points) The cost of living adjustment (COLA) is used by the government to ensure that the purchasing power of benefits is not diminished by inflation of prices. Let t = years since 2014 and $C(t)$ = COLA in the year t. Values of C are tabulated below

t	0	1	2
$C(t)$	1.5%	1.7%	0

   (a) Approximate $C'(1)$ as the average rate of change for $0 \le t \le 2$.

   (b) Find coefficients a, b, and c so that $C(t) = at^2 + bt + c$ agrees with the points in the table. Then use this quadratic function to approximate $C'(1)$.

   (c) Describe in words what the meaning of $C'(1)$ is.

4. (12 points) Let $f(x) = \arctan(x)$ on $[1, \sqrt{3}]$.

   (a) Explain why the function satisfies the conditions of the Mean Value Theorem.

   (b) Find all values of c which satisfy the conclusion of the Mean Value Theorem.

5. (12 points) Consider the function

$$f(x) = \begin{cases} \dfrac{7 - 16^{\frac{1}{x}}}{1 + 16^{\frac{1}{x}}} & \text{if} \quad x \neq 0 \\ 7 & \text{if} \quad x = 0 \end{cases}$$

   (a) Is $f(x)$ continuous at $x = 0$? *Hint:* $\dfrac{d}{dx} b^x = \ln b \cdot b^x$

   (b) Prove that the equation $f(x) = 1$ has a solution in the interval $[2, 4]$.

6. (8 points) An equation of the form $p(t) = Ae^{-ct} \sin(\omega t + \delta)$ represents the position of a object at time t. Find the velocity and acceleration of the object.

7. (8 points) For what values of a and b is $(1, -1)$ and inflection point on the curve $f(x) = ax^4 + bx^3$?

8. (8 points) Evaluate the upper and lower approximating sums for $\displaystyle\int_0^{\pi} \cos(x) dx$ with $n = 4$ and $n = 8$.

9. (8 points) Consider the integral function $g(x) = \displaystyle\int_0^x (1 - t^2) e^{t^2} dt$

   (a) Where is $g(x)$ increasing?

   (b) Find g's inflection point.

10. (8 points) Derive the formula for Newton's method and use it to find x_2 if $x_1 = \dfrac{\pi}{2}$ and we're searching for a zero of $f(x) = x - 2\sin x$.