Math 1A–Spring '16 **Name (Print):** ______________________
Chapter 4 Test

Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (12 points) For each function, find the critical points and the intervals of increase and decrease.
   (a) $f(x) = 5x + 6x^{-1}$
   (b) $h(x) = \dfrac{x}{(3x^2 - 5)^{1/3}}$

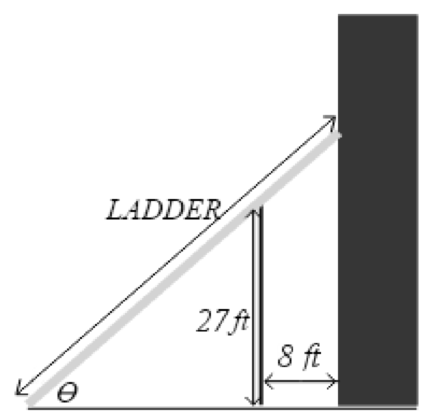
2. (12 points) Let $f(x) = 5x^3 - 3x$ on the interval $[-2, 2]$.
   (a) State why this function satisfies the conditions of the Mean Value Theorem.
   (b) Find all values $c \in (-2, 2)$ which satisfy the conclusion of the MVT.

3. (10 points) Suppose that $f(x)$ is differentiable for all x and that $2 \le f'(x) \le 6$ for all values of x. Show that $4 \le f(5) - f(3) \le 12$.

4. (20 points) Suppose $f(x) = \dfrac{20x}{x^2 + 25}$
   (a) What is the domain of f?
   (b) What intercept(s) does f have?
   (c) Describe the symmetry of f.
   (d) What asymptote(s) does f have?
   (e) Simplify $f'(x)$ and use it to find critical numbers and intervals of increase/decrease.
   (f) Simplify $f''(x)$ and use that to find inflection points of f.
   (g) Sketch a graph illustrating all these components of f.

5. (16 points) A wall 27 feet tall runs parallel to a tall building at a distance of 8 ft from the building as shown in the diagram (not to scale).

   We wish to find the length of the shortest ladder that will reach from the ground over the fence to the wall of the building.
   (a) First, find a formula for the length of the ladder, $L(\theta)$, in terms of θ. (*Hint*: split the ladder into 2 parts.)
   (b) To find the minimum length of the ladder, solve $L'(\theta) = 0$ and then use the critical value of θ_m to simplify $L(\theta_m)$.



6. (16 points) Answer the following questions about approximation with Newton's method:
   (a) Starting with an initial value of 3, use two iterations of Newton's method to approximate a zero of $f(x) = x^3 - 4x^2 - 2x + 14$. Simplify.
   (b) Choose $x_0 = 4$ to be an initial approximation of $\sqrt{13}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop x_1, a better rational approximation of $\sqrt{13}$; also give an arithmetic expression for the better approximation x_2 arising from a second step of Newton's method.

7. (14 points) Compute the limit:
   (a) $\displaystyle\lim_{t \to 0} \frac{t - \sin t}{t^3}$
   (b) $\displaystyle\lim_{x \to \infty} xe^{1/x} - x$