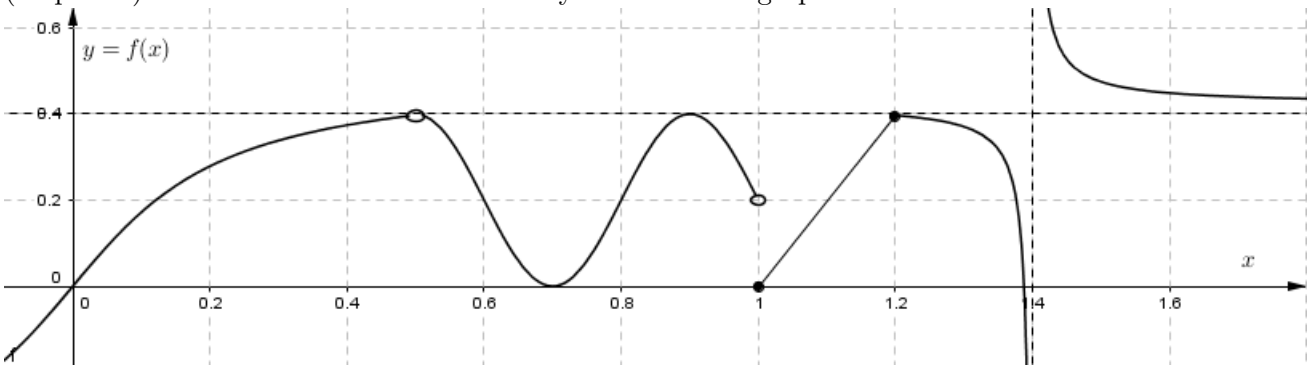
Math 1A **Name (Print):** _____
Final Exam

Write all responses on separate paper. Show your work in detail for credit. No calculators.

1. (12 points) Consider the function defined by its schematic graph below.



   (a) Find each limit or right "DNE" if the limit does not exist.

   i $\lim_{x\to 0.5} f(x)$ ii $\lim_{x\to 1^-} f(x)$ iii $\lim_{x\to 1} f(x)$

   (b) Find all the discontinuities and classify each as either a removable discontinuity, a jump discontinuity or a vertical asymptote.

   (c) Find $f'(1.1)$.

   (d) Solve for x: $f'(x) = 0$.

2. (12 points) Use the **definition** of the derivative (that is, $f'(x) \equiv \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$) to show that $\dfrac{d}{dx}x^3 = 3x^2$.

3. (12 points) Suppose that $g(2) = 3, g(3) - 4, g'(3) = 1$, and $g'(2) = -1$. Evaluate $\dfrac{d}{dx}g(g(x))\Big|_{x=2}$.

4. Let $f(x) = x^3 - x^2$ on $[0, 2]$.

   (a) Explain why the function satisfies the conditions of the Mean Value Theorem.

   (b) Find all values of c which satisfy the conclusion of the Mean Value Theorem.

5. (16 points) (a) Find each limit, or explain why it does not exist.

6. (10 points) Let

$$f(x) = \begin{cases} x - 1 & : x \le 2 \\ ax^2 + 3 & : 2 < x \le 3 \\ x^3 + b & : 3 < x \end{cases}$$

   Find values of a and b so that f is a continuous function.

7. (12 points) Consider $f(x) = \dfrac{1}{x+1}$

   (a) Use the definition of the derivative to find the derivative function $f'(x)$

   (b) Find an equation for the line tangent to the function at $x = 2$

8. (14 points) Use the intermediate value theorem to show that the equation $3^x = x^3$ has a solution for $0 < x < 2.5$. First state the Intermediate Value Theorem, then show precisely how the premise is satisfied and what conclusion follows. *Hint:* $\sqrt{3} \approx 1.732$