Math 1A – Chapter 3 Test – Typical Problems Set

1. Use the definition of the derivative to compute each of the following limits. Follow the form of this example: $\lim_{x\to 2}\frac{x^{10}-1024}{x-2}=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=f'(2)=10(2)^9=5120$

   a. $\lim_{x\to 0}\frac{(x^6+x+1)-1}{x}$ b. $\lim_{x\to 1}\frac{2^{3x}-8}{x-1}$ c. $\lim_{x\to 1/3}\frac{2\cos(\pi x)-1}{x-\frac{1}{3}}$

2. Find equations of the lines tangent to the curve $y=\frac{x}{1+x^2}$ which are parallel to the line $y-0.48x=0$. Sketch a graph illustrating these tangencies.

3. Newton's law of gravitation says that the magnitude *F* of the force exerted by a body of mass *m* on a body of mass *M* is

$$F=\frac{GmM}{r^2}$$

   a. Assume the bodies are in motion. Find *dF*/*dr* and write a sentence of two explaining what that means.

   b. Suppose that planet Xorkon attracts an object with a force that decreases at a rate of 3 N/km when *r* = 10,000 km. How fast does the force change when *r* = 5000 km?

4. Use the *definition* of the derivative to simplify $\frac{d}{d\theta}\cos(2\theta)$.

5. For what values of *x* does the graph of $f(x)=x^3+6x^2+x+4$ have a horizontal tangent?

6. A curve *C* is defined by the parametric equations $\begin{matrix} x=\sin(2t) \\ y=2\cos(t) \end{matrix}$.

   a. Show that *C* has two tangent lines at the origin: $(x,y)=(0,0)$ and find their equations.
   b. Find the points where the tangent line in the *x*-*y* plane is vertical.

7. Find an equation for the line tangent to $x^2y+xy^2=2xy$ at (1,1).

8. Let $y=\sqrt[3]{x}$.
   a. Find the differential *dy*.
   b. Evaluate *dy* and Δy if *x* = 8 and *dx* = Δx = 0.1
   c. Estimate $\sqrt[3]{8.1}$ using the line tangent to $y=\sqrt[3]{x}$ at (8,2). What is the relative error in your estimation?

9. Find the derivative of the function $f(x)=(\cos x)^x$ by first differentiating $\ln y$.

10. Consider $f(r)=2\sqrt{r}-3\sqrt[3]{r}$.
    a. Simplify formulas for the first and second derivatives of *f*(*r*).
    b. Use calculus to find the coordinates of the inflection point for *f*(*r*).

11. Find equations for the tangent lines to $y=\dfrac{x+1}{x-1}$ that are parallel to the line $x+2y=17$.

12. If $h(9) = 3$ and $h'(9) = -7$, find $\dfrac{d}{dx}\left(\dfrac{h(x)}{x}\right)\Big|_{x=9}$

13. In a fish farm, a population of the fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation

$$\frac{dP}{dt}=r_0\left(1-\frac{P(t)}{P_{\max}}\right)P(t)-H\cdot P(t)$$

where r_0 is the birth rate of the fish, $P_{\max}$ is the maximum population the pond can sustain and H is proportion of fish harvested in a year. If the pond can sustain a maximum population of 5,000 fish, the birth rate is 4% and the harvesting rate is 2%, what
(non-zero) population level(s) will not change, according to the model?

14. The gas law for an ideal gas at absolute temperature T (in kelvins), pressure P (in atmospheres), and volume V (in liters) is $PV = nRT$, where n is the number of moles of the gas and $R = 0.0821$ is the gas constant. Suppose that, at a certain instant, $P = 8$ atm and is increasing at a rate of 0.14 atm/min and $V = 11$L and is decreasing at a rate of 0.17 L/min. Find the rate of change of T with respect to time at that instant if $n = 10$ moles. Round your answer to four decimal places. $\dfrac{dT}{dt}\approx$? K/min

15. Use the definition of the derivative to compute $\dfrac{d}{dx}\sec(x)$.

16. Show that the curve described by the parametric equations $\begin{array}{l} x(t)=1-\cos(\pi t) \\ y(t)=\sin 3\pi t \end{array}$

has two tangent lines where $t = 1/3$ and find their equations. Illustrate these in a graph.

17. If $\sqrt{3}\sin x\cos y=1$, find a formula for $\dfrac{dy}{dx}$ using implicit differentiation.

18. Find $\dfrac{d^{63}}{dx^{63}}(x\sin x)$ by finding the first few derivatives and observing the pattern that occurs.

19. Use logarithmic differentiation to to find the derivative of $y=(\sec x)^x$

20. Verify the given linearization $\ln|x+1|\approx x$ near $x = 0$. What interval for x for is the linear approximation accurate to within 0.1?

21. Find an equation for the line tangent to $y=\tan^{-1}(\sin x)$ where $x = 0$.

22. Find an equation of the tangent line to the parametric curve $x = e^{\sqrt{t}},\quad y = -t^6 + t$ at the point corresponding to $t = 1$.

23. Find y'' if $x^8+y^8=1$.

24. Find an equation for the line tangent to the curve $y^3 + 3x^2 y + x^3 = 15$ at (1,2).

25. Use implicit differentiation to find an equation of the tangent line to the curve
$3(x^2 + y^2)^2 = 100(x^2 - y^2)$
at the point (4, 2). Give your answer in the form $y = mx + b$

26. At what point on the curve $y = 12x^{2/3} - 4x$ is the tangent line horizontal?

27. Derive a simple formula for the derivative $\frac{d}{dx}(\sinh^{-1} x)$

28. Find the points on the hyperbola $x^2 - 2y^2 = 1$ where the tangent line has slope = 2.

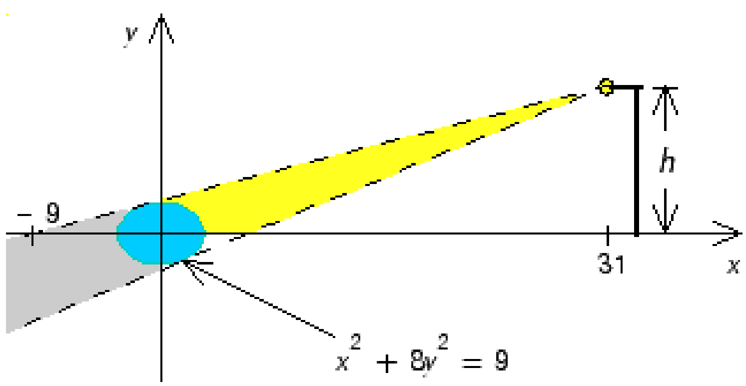
29. Let $y = \frac{\ln 8x}{x^8}$.

a. Find $y'(x)$

b. Find $y''(x)$

30. Find a parabola that passes through (1,10) and whose tangent lines at x = –2 and x = 1 are –5 and 7, respectively.

31. The figure shows a lamp located 31 units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 8y^2 \le 9$. If the point $(-9,0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?



32. Find an equation for the line tangent to the curve described by the parametric equations

$$x = \sin(3t) - \cos t$$
$$y = \cos(3t) - \sin t$$

where $t = \frac{\pi}{6}$.

33. Find y' if $x^{8y} = y^{7x}$.

34. Use a linear approximation (or differentials) to estimate $\frac{1}{99996}$.

35. A particle moves on a horizontal line so that its coordinate at time *t* is $x = e^{-t/4}\cos(2t)$.

a. Find the velocity and acceleration functions.
b. Find the distance the particle travels in the time $0 \le t \le \pi$.
c. When is the particle speeding up? When is it slowing down?

36. Consider $f(x) = \sqrt[3]{999 + x^3}$

a. Find the linearization of at $x = 9$ and use it to approximate $\sqrt[3]{1999}$.
b. What is the % error in your approximation?

37. Find the coordinates of the points where the line through (0,3) is tangent to the unit circle, $x^2 + y^2 = 1$.