Math 15 - Chapters 1 and 2 Test

Do any 14 of the following 20 problems. Due Monday, 3/19/12.

1. Consider the declaratives statements,
   P: The temperature outside is over $50°C$
   Q: John will fry an egg
   R: Mary is hungry
   Using logical connectives, write a proposition which symbolizes the following:
   (a) If it is over $50°C$ and Mary is hungry, then John will fry an egg on the side walk.
   (b) John will fry an egg on the side walk only if the temperature outside is over $50°C$
   (c) The temperature outside is not over $50°C$.
   (d) If John does not fry an egg on the side walk, then either Mary is not hungry or it is not over $50°C$.

2. Determine which of the following statements is true.
   _ Exactly one of these statements is false.
   _ Exactly two of these statements are false.
   _ Exactly three of these statements are false.
   _ Exactly four of these statements are false.
   _ Exactly five of these statements are false.
   _ Exactly six of these statements are false.
   _ Exactly seven of these statements are false.
   _ Exactly eight of these statements are false.
   _ Exactly nine of these statements are false.
   _ Exactly ten of these statements are false.

3. Construct a truth table for the proposition $P = [p \to (q \vee r)] \wedge [\neg(p \leftrightarrow \neg r)]$.

4. Is $\neg R \wedge (T \leftrightarrow P \leftrightarrow T \vee L)P$ a well formed expression? If not, how could you introduce new symbols into the express to make it a well-formed expression?

5. Suppose we know A is true, $A \to (B \to C)$ is true and that $B \to D$ is true. Can we conclude that $\neg C \to D$? Explain…or, that is, prove your claim.

6. Given that $\neg A \to B$ and $B \to A$ and $A \to \neg B$ can we conclude $A \wedge \neg B$? Use the method of indirect proof to prove or disprove.

7. Prove that implication is not associative.

8. What are the axioms of the propositional logic?

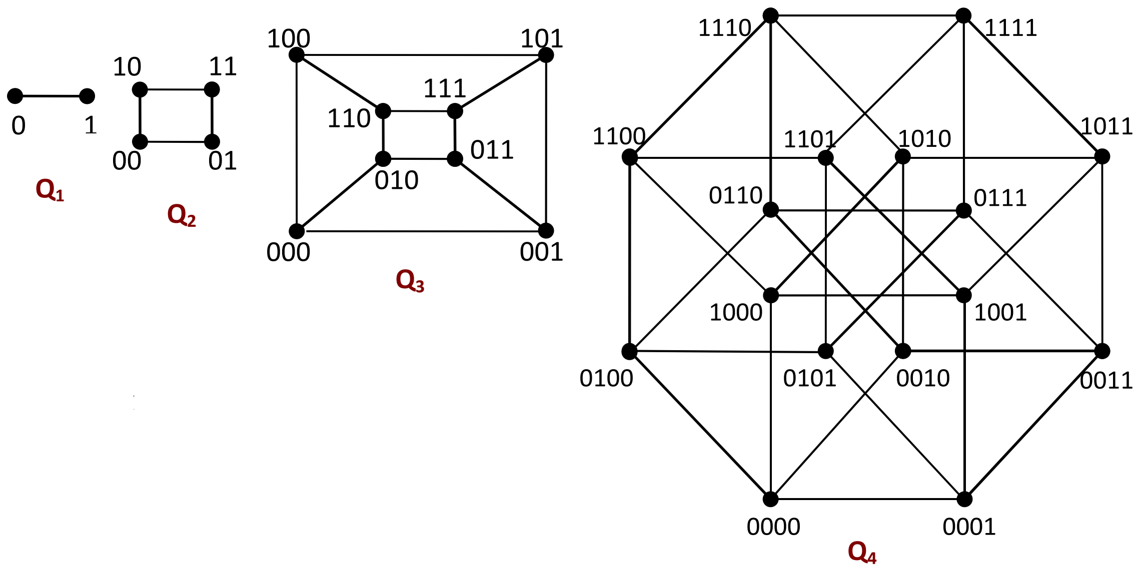
9. Prove or disprove: $(A \to B) \vee (A \to C) \to B \vee C$ is equivalent to $A \vee B \vee C$.

10. First, translate the following argument into propositional logic. Then, prove that the argument is valid using the method of formal derivation. Explain your answer.
Sally is destined to be either a fearless adventurer or a great psychiatrist. If Robert is not paranoid then Sally is not destined to be a great psychiatrist. Yet Sally is clearly not destined to be fearless adventurer. So Robert is definitely paranoid.

11. The handshaking principle states that in any graph, the sum of all vertex valences is twice the total number of edges. Write a sentence of two to explain why each of the following is a consequence of the handshaking principle with *n* people.
    a. The sum of all valences is an even number.
    b. In any graph, the number of vertices with odd valence is even.
    c. If all vertices in a graph have the same valence, *r*, then the graph has $\frac{1}{2}nr$ edges.

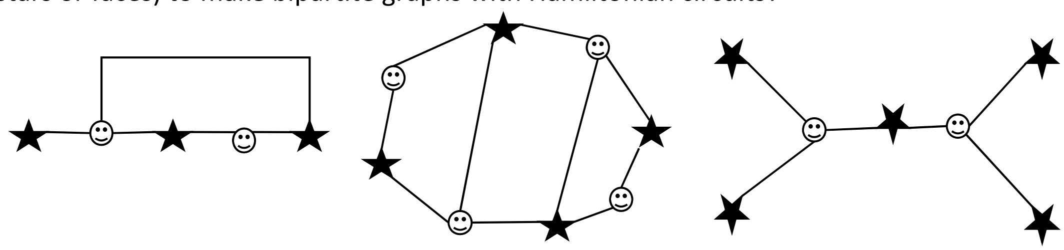
12. Draw graphs satisfying each of the following specifications:
    a. 6 vertices: 2 of valence 3, 2 of valence 4 and 2 of valence 5.
    b. 6 vertices: 1 of valence 2, 3 of valence 4 and 2 of valence 5.
    c. 6 vertices: 1 of valence 3, 4 of valence 4 and 1 of valence 5.

13.

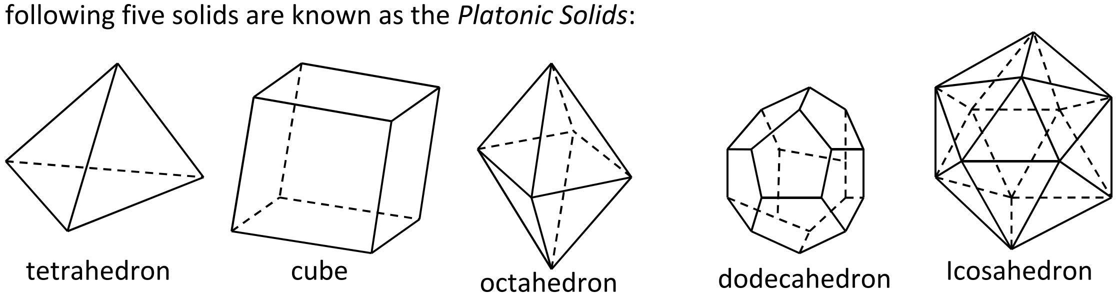


Of particular importance in coding theory are cube graphs, which may be constructed by taking as vertices all binary words (sequences of 0s and 1s) of a given length and joining two of these vertices if the corresponding binary words differ in just one place. The graph obtained in this way from the binary words of length *k* is called the ***k*-cube** (or *k-dimensional* cube), and is denoted $\boldsymbol{Q_k}$. Cube graphs for *k* = 1,2,3 and 4 are shown above. Give a formula for the number of edges of a $\boldsymbol{Q_k}$ graph.

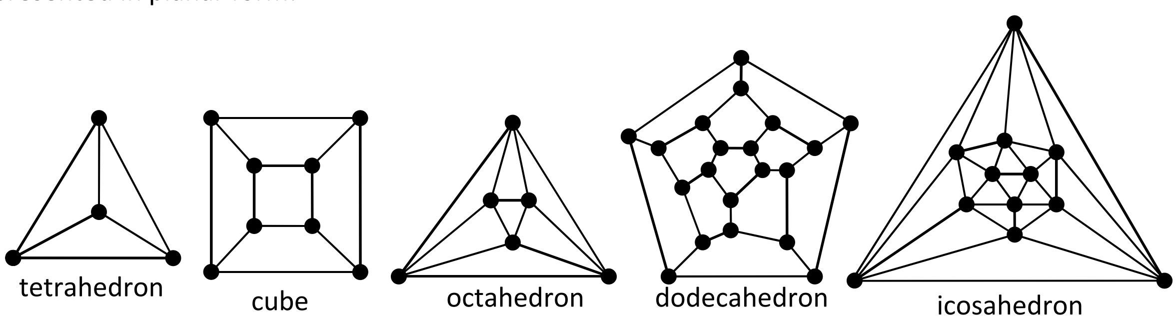
14. A **bipartite graph** is a graph whose vertex-set can be split into sets *A* and *B* such that all edges in the graph join a vertex from *A* with a vertex from *B*. Under what condition will a bipartite graph have a Hamiltonian circuit? How would you add vertices and edges to the graphs below (vetices are either stars or faces) to make bipartite graphs with Hamiltonian circuits?



15. The following five solids are known as the *Platonic Solids*:



If you consider the edges and vertices of these solids as the edges and vertices of a graph, each can be represented in planar form:

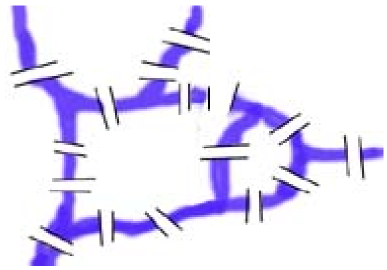


a. Which of the platonic graphs have an Euler circuit?

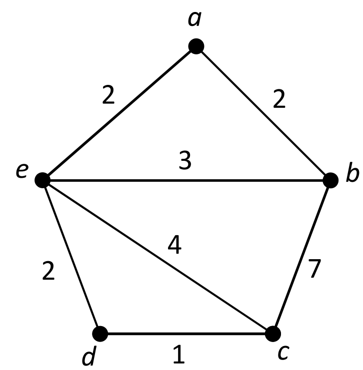
b. Which of the platonic graphs have a Hamiltonian circuit?

c. Euler's formula applies to polyhedra. It states that if *n*, *m* and *f* are the number of vertices, edges and faces, respectively, then $n - m + f = 2$. For example, for the tetrahedron, $4 - 6 + 4 = 2$. Verify Euler's formula for the other 4 platonic solids.

16. The town of Palmberg has rivers, bridges and islands as shown in the diagram.
    a. Model the map as a graph where each separate land mass is represented by a vertex and each bridge is an edge.
    b. Is there a there a way the people of Palmberg can walk along a path that will pass over each bridge exactly once? Explain.



17. A postman wishes to deliver mail along all the streets in his area and then return to his post office. The graph of the streets (edges) and intersections (vertices) is shown at right. How can the route be planned so as to minimize the total distance traveled?



18. How many nonisomorphic rooted trees are there with 5 vertices?

19. Consider the divisibility poset {1, 2, 5, 20, 50, 100}, | .
    a. Draw the Hasse diagram of this poset.
    b. Determine whether this poset is a lattice.

20. Prove or disprove: It is possible to inscribe a circle in any convex quadrilateral in which the sums of the opposite sides are equal.