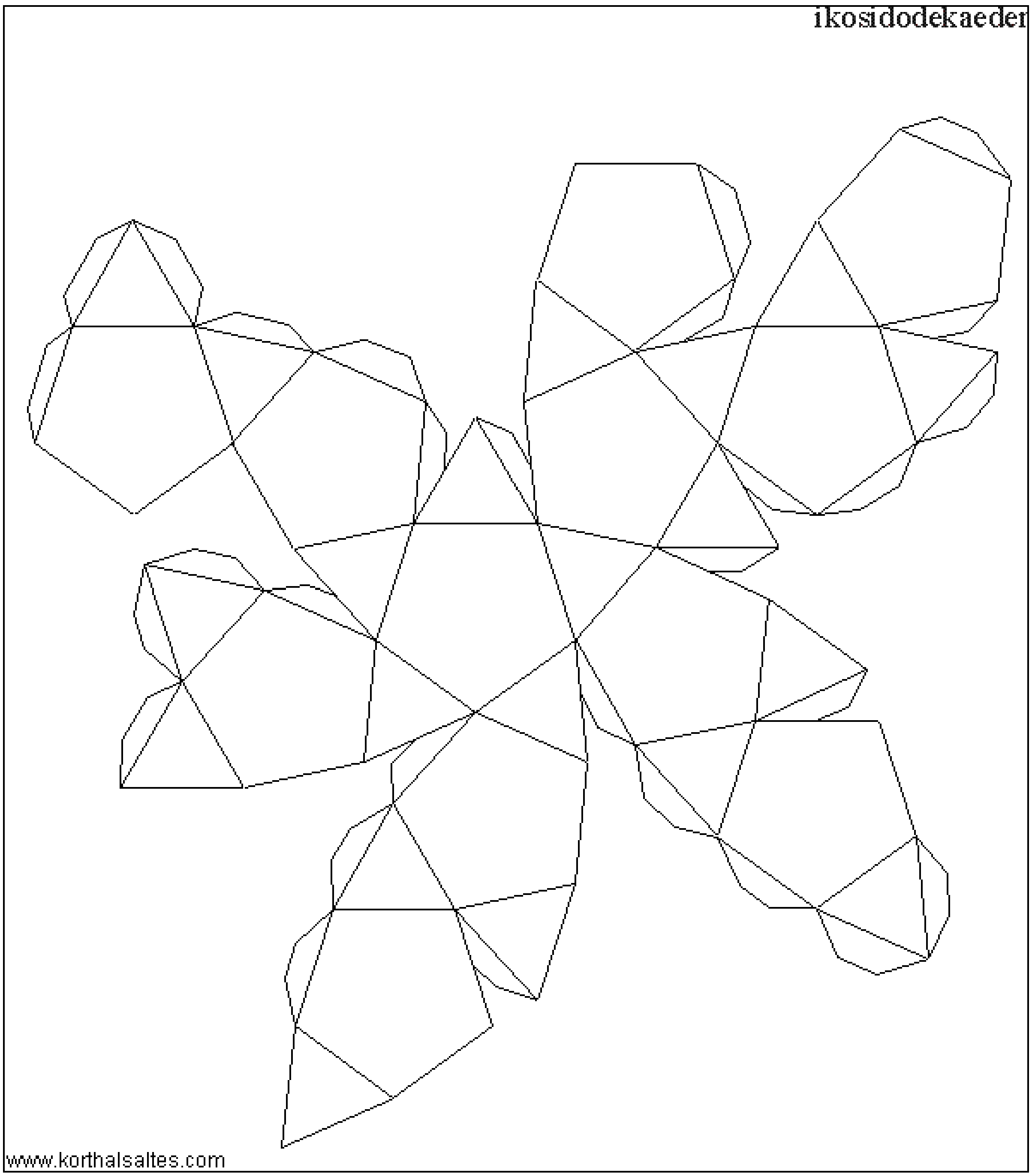
Math 15 – Discrete Structures – §2.6 – Homework 7 Solutions

2.6#12: What is the fewest number of colors needed to color K_n such that no two vertices of the same color are joined by an edge?
ANS: Since each vertex is connected to each other vertex, you'd need n colors to accomplish this.

2.6#14: Recall the definition of $K_{n,m}$, the complete bipartite graph with n vertices in one group and m in the other. What is the fewest number of colors needed to color $K_{n,m}$ such that no two vertices of the same color are joined by an edge?
ANS Two colors will do. Color all the n vertices in one group, say red, and all the vertices in the other group, say, green. That'll do.

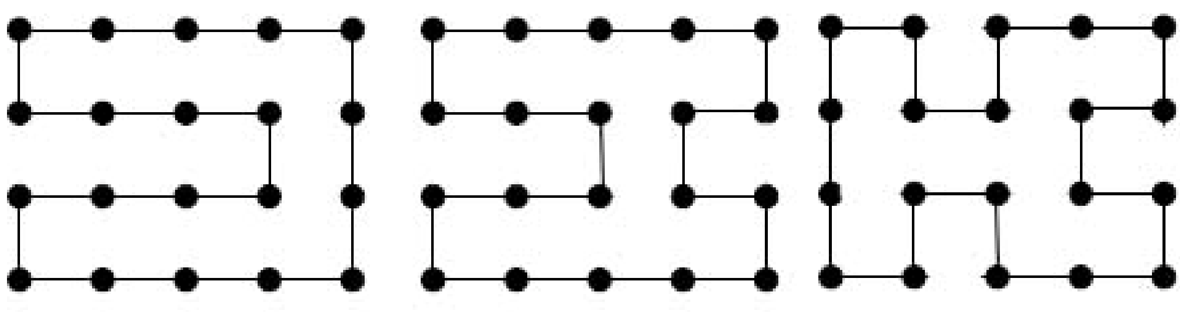
2.6#16: An icosidodecahedron can be constructed from paper using the following cutout:



How would you "truncate" this to produce the great rhomibcosidodecahedron?

A truncated icosidodecahedron (also known as a “great rhombicosidodecahedron” or “omnitruncated icosidodecahedron”) is a polyhedron with 120 vertices. You can view various models for this at http://mathworld.wolfram.com/GreatRhombicosidodecahedron.html .   Each Each vertex looks the same: a square, a hexagon, and a decagon come together at each vertex. How many edges does the icosidodecahedron have?
ANS: Theorem 2.6 states, “Let *G* be an undirected graph. The sum of the degrees of the vertices of *B* equals twice the number of edges in *G*.” Consider the graph *G* formed by the edges and vertices of the rhombicosidodecahedron. Since each vertex has degree 3 and there are 120 of them, the sum of degrees is 360, so there are 180 edges.

2.6#24: Find a Hamiltonian circuit for the following graph.
ANS: Here are three of the possible Hamiltonian circuits. Can you find more? How many are there altogether? How many Hamiltonian circuits on an *m* by *n* grid?



2.6#30: Let *G* be a connected, undirected graph. Prove that there is a subgraph *T* of *G* such that *T* contains all the vertices of *G*, and *T* is a tree. (Such a subgraph is called a spanning tree.) Give a constructive proof that explains how to construct a spanning tree of a graph.

Proof. Keep pruning edges from circuits until there are no circuits.