Math 15 – Discrete Structures – 2.4 Homework 5 Solutions

2.4#33: The multiplication table for $\mathbb{Z}/11$ is

$\cdot$	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	1	3	5	7	9
3	0	3	6	9	1	4	7	10	2	5	8
4	0	4	8	1	5	9	2	6	10	3	7
5	0	5	10	4	9	3	8	2	7	1	6
6	0	6	1	7	2	8	3	9	4	10	5
7	0	7	3	10	6	2	9	5	1	8	4
8	0	8	5	2	10	7	4	1	9	6	3
9	0	9	7	5	3	1	10	8	6	4	2
10	0	10	9	8	7	6	5	4	3	2	1

2.4#36: Is `0060324814` a valid ISBN value?

ANS: No. $1\cdot 0+2\cdot 0+3\cdot 6+4\cdot 0+5\cdot 3+6\cdot 2+7\cdot 4+8\cdot 8+9\cdot 1+10\cdot 4$
$$= 18+15+12+28+64+9+40$$
$$= 186 \equiv 10(mod 11)$$

2.4#37: Show that a check digit will always detect an error of swapping a pair of adjacent digits.

ANS: Suppose the *k*th and (*k*+1)st digits in an ISBN are swapped. Then the only terms in the check digit sums which are different are $kd_k+(k+1)d_{k+1}$ in the first and $kd_{k+1}+(k+1)d_k$ in the second. In order for an error to go undetected, the difference between these would have to be divisible by ll, that is, $kd_k+(k+1)d_{k+1}-(kd_{k+1}+(k+1)d_k)=d_{k+1}-d_k=11n$, for some integer, *n*. But $0\le d_k, d_{k+1}\le 10$ so the difference between them is between 0 and 10, so the difference must be 0, that is, $d_k=d_{k+1}$.

2.4#38: Show that the check digit will always detect an error of changing a single digit.

ANS: Suppose the digit a_k is replaced by $\hat{a}_k \neq a_k$. The checksum will change if the difference
$$1a_1+2a_2+\cdots+ka_k+\cdots+10a_{10}-(1a_1+2a_2+\cdots+k\hat{a}_k+\cdots+10a_{10})$$
is divisible by 11. That is, $k(a_k-\hat{a}_k)=11n$, for some integer, *n*. But *k* is between 1 and 10, and both a_k and $\hat{a}_k$ are between 0 and 10. As the multiplication table from #33 above shows, the product of two numbers is a multiple of 11 only if one of those numbers is 0, and since $k\neq 0$, we must have $a_k-\hat{a}_k=0$.

2.4#39: Unfortunately, there were too many books and not enough ISBNs; effective January 2007, ISBN numbers must be 13 digits long. The check digit scheme for 13-digit numbers is different. Explain why the obvious modification to the old system won't work. That is, find a 12-digit string $a_1a_2\cdots a_{12}$ where the quantity $\left(\sum_{i=1}^{12} i\cdot a_i\right) \bmod 14$ doesn't change after changing a single digit.

ANS: Note that 2*7 = 14 = 0mod14, so if the 7^{th} digit is changed from a 2 to a 4, the checksum will be the same. 000000200000 and 000000400000 both have checksum 0. Also, increasing an even entry with digit 2 to a 9 will not change the checksum. 123456789012 and 123456789019 both have checksum 0.

2.4#40: Will the "obvious modification" in Exercise 39 detect the error switching two adjacent digits?

ANS: Yes. The thing is that if the digits are adjacent then one coefficient will be odd and the other even. As was argued in #37 above, the difference between the checksum values when adjacent digits are swapped is $d_{k+1}-d_k=14n$, but they can't differ by more than 13.