Math 15 – Discrete Structures – 1.4 & 1.5 Homework Solutions

<u>1.4#22</u>: In 4-point geometry, do triangles exist? In other words, is it possible to have three distinct points, not on the same line, such that a line passes through each pair of points? Why or why not?

ANS: Yes. There are four points (Axiom 3), and three can't be on the same line (Axiom 4). Every pair of distinct points determines a line (Axiom 1), so any three points and the three lines they determine will form a triangle.

<u>1.4#24</u>: Consider the following model for 4-point geometry: Points: 1,2,3,4

Lines

[1 2] [1 3] [1 4] [2 3] [2 4] [3 4]

A point "is on" a line if the line's box contains the point.

(a) Give a pair of parallel lines in this model (Recal to the definition: Two lines, *l* and *m*, are <u>parallel</u> if there is no point *x*, such that *x* is no *l* and *x* is on *m*.

ANS: [1 2] [3 4] are parallel. So are [1 4] and [2 3] .

(b) [1 2] and [1 3] are intersecting.

<u>1.4#26</u>: Recall the Badda-Bing axiomatic system:

Undefined terms: badda, bing, hit

Axioms:

1. Every badda hits exactly four bings
2. Every bing is hit by exactly two baddas
3. If *x* and *y* are distinct baddas, each hitting bing *q*, then there are no other bings hit by both *x* and *y*.
4. There is at least one bing.

Consider the following definition in this system: Let *x* and *y* be distinct baddas. We say that a bing *q* is a <u>boom</u> of *x* and *y*, if *x* hits *q* and *y* hits *q*.

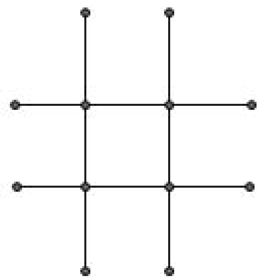
Rewrite axiom 3 using this definition.

ANS Two distinct baddas can have at most one boom. For "boom" read, "common vertex."

<u>1.4#28</u>: Describe a different model for the Badda-Bing axiomatic system, using squares and vertices, where each square is the same size.

ANS: This would be an infinite checkerboard.

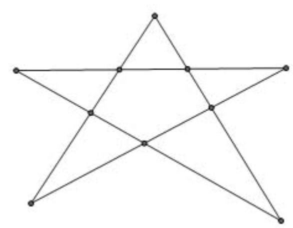
1.4#29: In the Badda-Bing axiomatic system, let a "badda" be a line segment and let a "bing" be a point, and say that a line segment "hits" a point if it passes through it. In the diagram below, there are 4 baddas and 12 bings. Is this a model for the system? Which of the axioms does this model safisfy? Explain.



Every line passes through four points (Axiom 1), any two distinct lines intersect in at most one point (Axiom 3), and there is at least one point (Axiom 4). However, Axiom 2 fails because there are some points that lie on only one line.

1.4#30: Describe a model for the Badda-Bing axiomatic system with 10 bings, where a "badda" is a line segment and a "bing" is a point.

1. Every line hits exactly four points.
2. Every point is hit by exactly two lines.
3. If *x* and *y* are distinct lines, each hitting point *q*, then there are no other points hit by both *x* and *y*.
4. There is at least one point.



1.5#20: Let x and y be real numbers. If x is rational and y is irrational, then $x + y$ is irrational.
Proof (by contradiction): Suppose $x + y$ is rational. Then by definition, there exist integers a and b such that $x + y = \frac{a}{b}$. Since x is rational, there exist integers c and d such that $x = \frac{c}{d}$. Substituting, we have $\frac{a}{b} + y = \frac{c}{d}$. Subtracting x from both sides, then $y = \frac{c}{d} - \frac{a}{b} = \frac{cb-ad}{bd}$. Now, since the integers are closed under the operations of addition and multiplication, the numerator and denominator of y are integers, contradicting the assumption that y is irrational. Thus, by reductio ad absurdum, $x + y$ is irrational.

1.5#22: Recall the badda-bing axiomatic system. Prove that if *q* and *r* are distinct bings, both of which are hit by baddas *x* and *y*, then $x = y$.
Proof (by contradiction): Suppose $x \neq y$ are baddas that each hit distinct bings *q* and *r*. This contradicts axiom 3, since you would have distinct baddas hitting the bing *q* and so "there are no other bings hit by both *x* and *y*."

1.5#24: In the axiomatic system for four-point geometry, prove the following assertion using a proof by contradiction.

> Suppose that *a* and *b* are distinct points on line *u*. Let *v* be a line such that $u \neq v$.
> Then *a* is not on *v* or *b* is not on *v*.

Proof (by contradiction): Suppose that *a* and *b* are distinct points on line *u*, and that *v* is a different line containing those two points, this would be a contradiction to axiom 1, that every pair of distinct points determines a line.