Math 15 – Discrete Structures – §4.2 – Homework 13 Solutions

<u>4.2#18</u>: A certain brand of jellybeen comes in four colors: red, green, purple, and yellow. These jellybeans are packaged in bags of 50, but there is no guarantee as to how the colors will be distributed; you might get a mixture of all four colors, or just some red and some green, or even (if you are lucky) a whole bag of purple.
(a) Explain how to view the color distribution of a bag of jellybeans as combination.
ANS: We need three separators to group the jellybeans into a partition of 50.
(b) There are $\binom{53}{3} = \frac{53\cdot 52\cdot 51}{3\cdot 2} = 23426$ ways to position the 3 separators in the 53 positions, arranging same-colored jellybeans in between each separator (or on an end.)

<u>4.2#20</u>: How many ways are there to distribute 12 identical bones among three different dogs.
ANS: This is equivalent to the number of ways of arranging 12 0's and 2 1's. Think of the 1's as separating the 0's into three groups, one group for each dog. Thus there are $\binom{14}{12} = \frac{14\cdot 13}{2} = 91$ different ways to do this.

<u>4.2#22</u>: The streets of many cities are based primarily on a rec5tangular grid. In such a city, if you start at a given street corner, how many different ways are there to walk directly to the street corner that is 5 blocks north and 10 blocks east?
ANS: To complete this path, one must travel north (N) 5 blocks and east (E) 10 blocks. So this is the number of "words" composed of 5 N's and 10 E's. To specify such a word is equivalent to choosing which of 15 positions to put the 10 E's, which is $\binom{15}{10} = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11}{5\cdot 4\cdot 3\cdot 2} = 3003$

<u>4.2#28</u>: Compute the coefficient of z^4 in the expansion of $(2z+5)^7$.
ANS: $(2z+5)^7 = \sum_{k=0}^{7}\binom{7}{k}(2z)^k 5^{7-k} = 5^7 + \cdots + \binom{7}{4}(2z)^4 5^3 + \cdots + (2z)^7$. So we see that the term with z^4 is $\binom{7}{4}(2z)^4 5^3 = 70000z^4$.

<u>4.2#30</u>: Pascal's triangle contains entries for $\binom{n}{k}$ for k = 0, 1, 2, … , n in each row starting with $\binom{0}{0} = 1$ at the top. Each entry is the sum of the two entries adjacent and above it, with 1's framing the boundary.
We can write this as a recurrence relation, $\binom{n}{k} = \begin{cases} 1 & \text{if } k=0 \text{ or } k=n \\ \binom{n-1}{k} + \binom{n-1}{k-1} & \text{if } n>k>0 \end{cases}$

You can think of the recurrence relation $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ as saying that the number of ways to choose k things from a collection of n things is to consider whether or not one of the particular elements of the n is chosen or not. If you're not going to choose that particular thing, then there are $\binom{n-1}{k}$ ways to choose k from the remaining n – 1 and if you are going to choose it then you need only choose k – 1 things from the remaining n – 1, which can be done in $\binom{n-1}{k-1}$ ways.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1