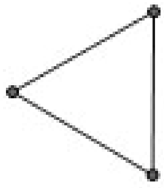
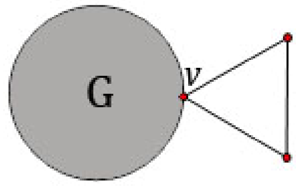
Math 15 – Discrete Structures – §3.5 – Homework 12 Solutions

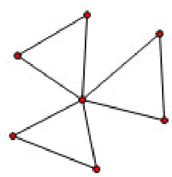
<u>**3.5#20**</u>: Define a TGraph as follows.

B: This is a TGraph: **R:** If G is a TGraph and v is a vertex of G then this is a TGraph:



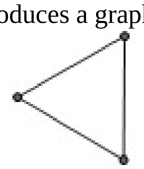


Draw an example of a TGraph with seven vertices.
ANS:

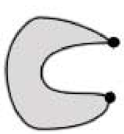
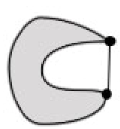


<u>**3.5#21**</u>: Prove that every vertex in a TGraph has even degree.
ANS: The simplest TGraph is a triangle where each vertex has degree 2. Suppose that *G'* is a TGraph that has all vertices with even degree. Then adding a triangle to a vertex will increase that vertex's degree by 2 and will add two new vertices, each with degree 2, so all the vertices have even degree. So, by induction, the recursive definition always produces a graph whose vertices all have even degree.

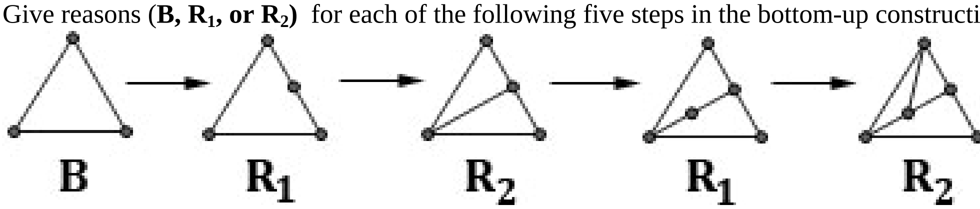
<u>**3.5#22**</u>: Prove by induction that any TGraph can be 3-colored.
ANS: The base case is a triangle which can be 3-colored. Suppose that *G'* is a TGraph that can be 3-colored. Then adding a triangle to a vertex produces another TGraph that can also be 3-colored by simply coloring the two new vertices the two colors not used by the color of the vertex the new triangle is attached to. So, by induction, the recursive definition always produces a graph whose vertices can be 3-colored.

<u>**3.5#23**</u>: Define a KGraph as follows. **B:**  is a KGraph.

R_1: If  is a KGraph, so is  (Any edge can be bisected.)

R_2: If  is a KGraph, so is  (Any two vertices can be joined.)

Give reasons (**B, R_1, or R_2**) for each of the following five steps in the bottom-up construction of a KGraph:



<u>**3.5#24**</u>: Prove that in any KGraph the number of edges is greater than or equal to the number of vertices.
ANS: In the base case the number of vertices = the number of edges = 3. Assume that the number of edges is ≥ the number of vertices in a KGraph, *G'*. Then R_1 will add one edge and one vertex, maintaining the inequality, while R_2 will add an edge and leave the number of vertices the same, also maintaining the inequality.