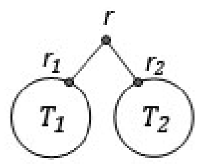
Math 15 – Discrete Structures – §3.3 – Homework 10 Solutions

<u>3.3#20</u>: Recall the recursive of a binary tree:

The empty tree is a binary tree.

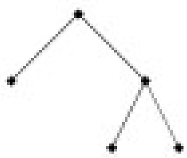
A single vertex is a binary tree. In this case, the vertex is the root of the tree.

If *T*1 and *T*2 are trees with roots *r*1 and *r*2, respectively, then the tree

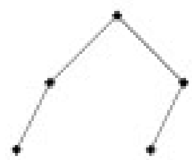


is a binary tree with root *r*. Here the circles represent binary trees *T*1 and *T*2 and if either of these trees is the empty tree, then there is no edge from *r* to that tree.

(a) Give an example of a full binary tree with five nodes.



(a) Give an example of a binary tree with five nodes that is not a full binary tree.



<u>3.3#22</u>: Let *G* be an undirected graph, perhaps unconnected. The different pieces that make up *G* are called the connected components of *G*. More precisely, for any vertex *v* in *G*, the connected component G_v containing *v* is the graph whose vertices and edges are those that lie on a path starting at *v*. Give a recursive definition for G_v.

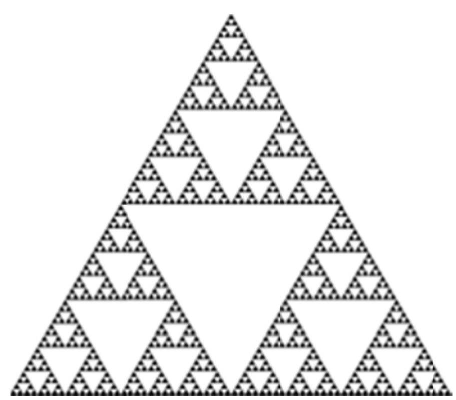
ANS:

B. *v* is in G_v.

R1. If *x* is a vertex in G_v, every edge touching *x* is also in G_v.

R2. If *e* is an edge in G_v, every vertex touched by *e* is also in G_v.

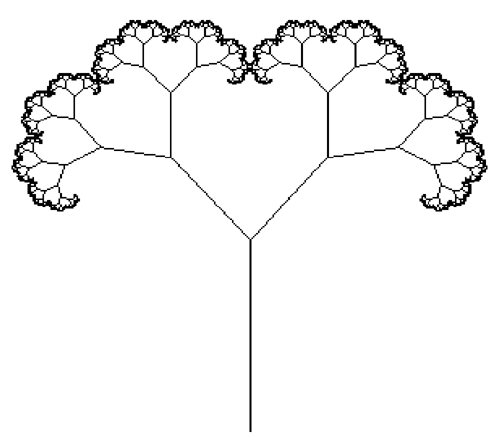
<u>3.3#24</u>: Give a recursive definition for where the is the Sierpinski gasket fractal shown at right.



ANS:

an equilateral triangle if $n = 1$. If $n > 1$ then is the arrangement of triangles formed by removing from each triangle *T* of the triangle formed by joining the midpoints of the sides of *T*.

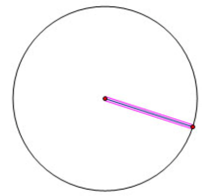
<u>3.3#26</u>: Give a recursive definition for , where is the fractal tree shown at right.



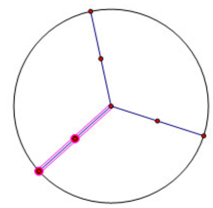
ANS: = a straight line segment with one end primed for growth. If $n > 1$ then is formed by appending two line segments to the growth ends of about half as long as the as the line segment they're grown from and forming an angle of about 75 degrees, with the unattached ends of these new segments being primed for growth.

3.3#28: First, here's a step-by-step guide for creating the iterated graphic in Geometer's Sketchpad. Give

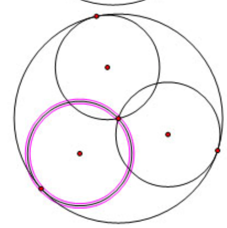
First draw a circle, draw its radius and select the radius resulting in something like the picture at right. Then on the "construct" menu choose construct the midpoint for the radius. Then deselect all and double click on the center of the circle to choose it as the center of rotation. Then select the radius, its midpoint and the point on the radius at the circumference and rotate this by 120 degrees twice. This is done using options on the "transform" menu.



You should now have something like the picture shown at right.

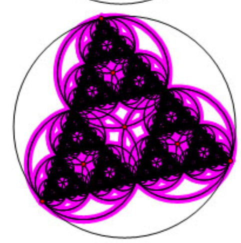


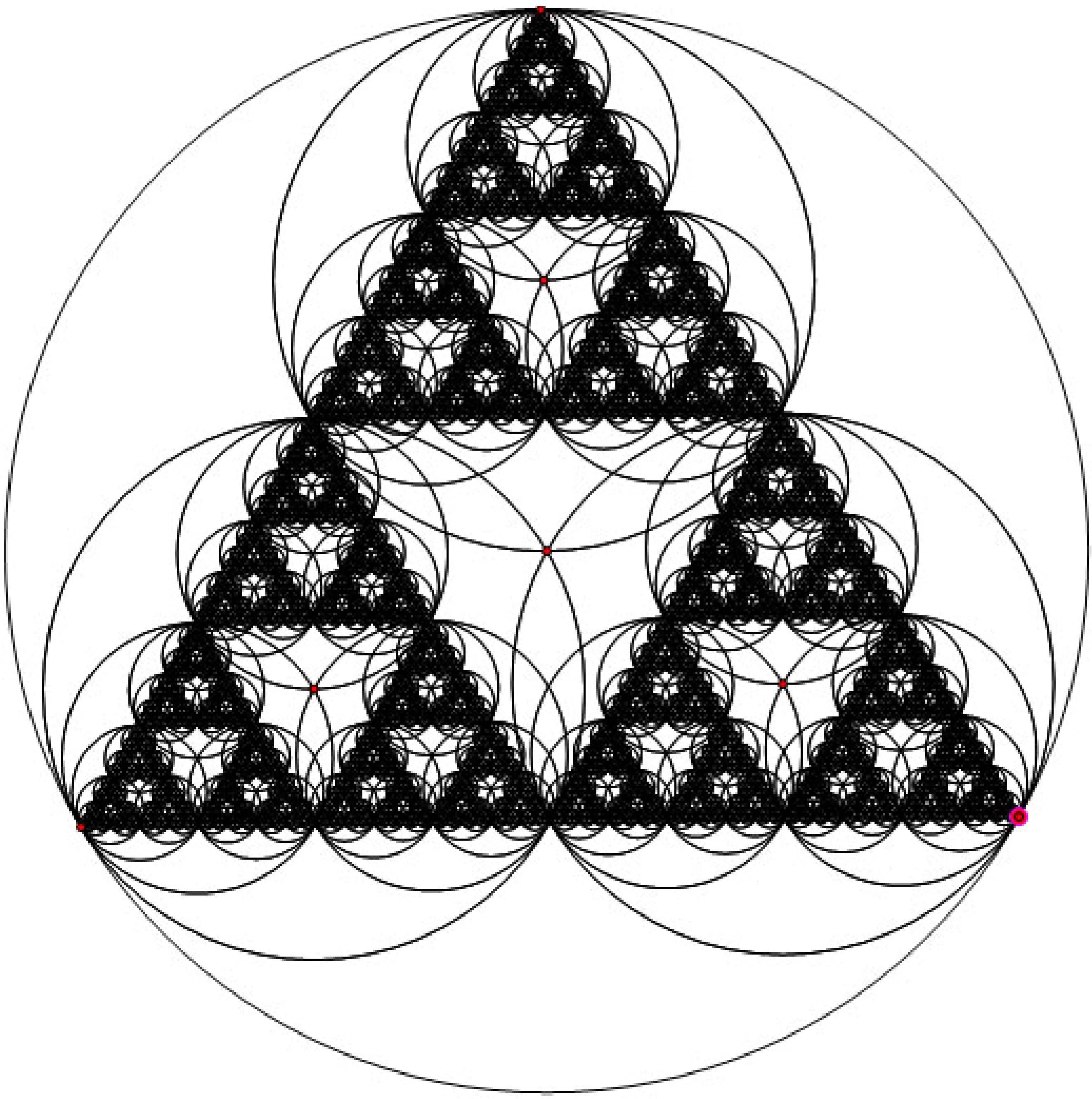
Deselect all and then select just the three radii and click ctrl+H to hide them. Then draw three circles centered at the midpoints of the three radii by dragging the circumferences to the point on the circumference of the larger circle where that radius meets it. You now have something like the picture shown next.



Now deselect all and choose the center of the big circle followed by the point on the circumference to the right (these were the original points used to create the big circle.)

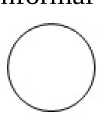
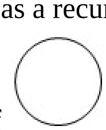
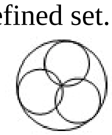
Now find that "iterate" on the "transform" menu is available and create three constructions, each time mapping the center of the big circle to a center of one of the three little circles and mapping the circumference point to the relative circumference point on the big circle. Indicate 7 iterations. You will get something like the following, which may adorn the next Math Field Day T-shirt, if there is ever another Math Field Day.





Oh…so, we still haven't properly answered the question? The question asks that we give a recursive definition for $C(n)$ where the sequence $C(1), C(2), C(3), \ldots$ is the fractal illustrated above.
ANS: $C(1)$ is a circle. For $n > 1$, $C(n)$ is formed from $C(n-1)$ by adding, for each circle S in $C(n-1)$, three circles, each with half the diameter of S and each passing through the center of S and touching the original circumference point or a point rotated 120 degrees from there.

3.3#29: Give an informal definition of the shape as a recursively defined set.

ANS: Base case:  $\in C$ Recursive step: If  $\in C$ then  $\in C$.