Note that if in (3) we allow n to be negative, it is still true that

$$a_n^3 + b_n^3 = c_n^3 + (-1)^n,$$

and these sequences are also given by Ramanujan [**5**, p.341].

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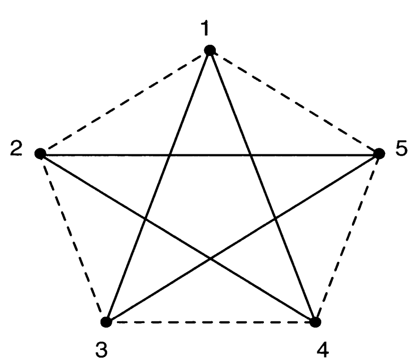
Ramsey's Theorem Is Sharp

SOLOMON W. GOLOMB
Department of Electrical Engineering Systems
University of Southern California
Los Angeles, CA 90089

Ramsey's Theorem [**1**] asserts that if the $\binom{6}{2} = 15$ edges of K_6, the complete graph on six points, are colored using two colors, there will be a *triangle* (a K_3 subgraph of K_6) with all three of its edges having the same color. This is sometimes called "the party problem," because if you select any six people at a party, it is guaranteed that either three of them will all know each other, or there will be three of them no two of whom know each other. To see the equivalence, represent each of the six people by a point, connect two points with a red line if the two people represented know each other, but by a blue line if they don't. Then by Ramsey's Theorem there will either be a solid red triangle (three mutual acquaintances) or a solid blue triangle (three mutual strangers).

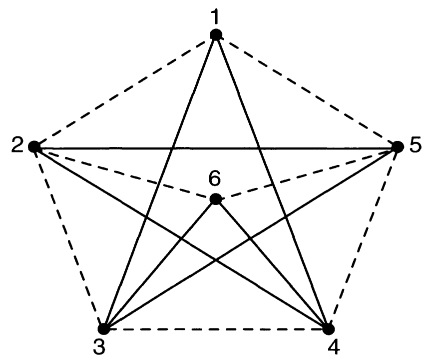
The purpose of this note is to present a visually striking proof that if any one of the 15 edges of K_6 is removed, the resulting graph (with 14 edges connecting the six points) *can* have all its edges colored, using two colors, without creating a solid-color triangle.

We first exhibit a two-coloring of the $\binom{5}{2} = 10$ edges of K_5 that creates no solid-color triangle:



(Our two colors are shown as solid and dotted lines. In this figure, there is a solid pentagon and a dotted pentagon, and clearly no single-color triangle.)

We now adjoin a *sixth* point in the *middle* of the K_5-figure just pictured, connecting it to the previous five points with two solid lines and two dotted lines.



Voila!

In addition to pentagons (5-cycles), there are now also quadrilaterals (4-cycles) in each of the two colors, but still no triangles (3-cycles). The only edge which is missing from the *complete* graph, K_6, on these six points is the edge from 1 to 6.

In terms of the party problem, if only one of the 15 pairs (among 6 people) refuses to acknowledge whether or not they are acquainted, we can no longer promise to exhibit a trio of mutual acquaintances or mutual strangers.

One generalization of Ramsey's original problem is: What is the smallest positive integer $R = R_c$ such that, if the complete graph K_R on R points have all $\binom{R}{2}$ of its edges colored in c colors, a solid color triangle is guaranteed to exist. The exact value of R_c is known for only a few small values of c, such as $R_2 = 6$ (the original Ramsey Theorem) and $R_3 = 17$.

For a context for $R_3 = 17$, suppose that in a certain high school class, each pair of students are either mutual friends, mutual enemies, or mutually indifferent. (While these relationships are symmetric, they are not assumed to carry over to third parties. The friend of a friend can be an enemy; the enemy of an enemy need not be a friend.) Then in any collection of 17 students there is certain to be a trio of either mutual friends, mutual enemies, or people who are mutually indifferent.

The following remarkable generalization of our triangle-free 2-coloring when one edge is removed from K_6 was observed by Herbert Taylor.

THEOREM. *For every $c \geq 2$, when a single edge is removed from the complete graph on R_c points, what remains can be c-colored without forming any solid-color triangles.*

Proof. We do not need to know the actual value of R_c to prove this theorem! By the definition of R_c, the complete graph on $R_c - 1$ points *can* have all its edges colored using c colors in such a way that no solid-color triangle is formed. Start with this coloring of the complete graph on $R_c - 1$ points. Designate any one of these points as P, and introduce a new point P' (the *clone* of P). Connect P', with edges, to each of the original points *except* P, and color the edge from P' to Q with the same color as the edge from P to Q, for every point Q except P and P'. If a solid-color triangle were formed, it would already have existed in the previous graph with P instead of P'.

Since there is no edge between P and P', there can be no triangle using both P and P'; so our new graph with R_c points lacks only the edge from P to P' to be the complete graph on R_c points, and it is edge-colored in c colors with no solid-color triangles. ■

Note that our illustration of a two-coloring of K_6 with one edge missing, having no solid-color triangles, is a special case of this general result, where the new point, 6, is the clone of 1.

Since K_{17} has $\binom{17}{2} = 136$ edges, we can 3-color 135 of these without forming a solid-color triangle!

We can also clone more than one point. For example, it is sufficient to remove only five of the 45 edges of K_{10} so that the remaining 40 edges can be 2-colored without forming a solid-color triangle. To achieve this, start with the triangle-free 2-coloring of the edges of K_5. We then clone each of the five original points of K_5, sequentially, adjoining one at a time, following the procedure in the proof of the Theorem. When we are done, the only edges missing from K_{10} are the five that connect each of the original points to their clones.

If we try to use this procedure when adjoining more than one clone to the same original point, all edges connecting the points in the same clone set must be omitted. For example, we can 2-color all but 15 of the 105 edges of K_{15} without forming a solid-color triangle, by adjoining two clones to each of the five original points of K_5.

When we use only one color, the Ramsey number R_1 is 3. (We can color the single edge of K_2, but not all three edges of K_3, using only one color, without forming a solid-color triangle.) The reader is encouraged to experiment with adjoining clones to the two original points of K_2, using only one color, and avoiding triangles, as just described. (What results are the complete bipartite graphs connecting the two clone sets, and these graphs are all triangle-free.)

Graham, Rothschild, and Spencer [**2**] give an extensive treatment of generalizations of Ramsey's Theorem.

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Where the Camera Was, Take Two

ANNALISA CRANNELL
Franklin & Marshall College
Lancaster, PA 17604
annalisa.crannell@fandm.edu

In the very nice article "Where the camera was" [**1**], Byers and Henle approximated the position of a photographer from geometric clues in an old photograph of John M. Greene Hall at Smith College. Here, we give an approach to the problem that is slightly more geometric.

We will make one simplifying assumption that the original article did not make: that the photo was not cropped, meaning that the center of the photograph was the center of the photographer's aim. Using the diagonals of the rectangle (see FIGURE 1), we can determine where to aim our own camera to best recreate the original photograph.