Math 15 - Spring 2017 - Homework 4.2 Solutions

1. (4.4 # 22) An urn contains three red balls, four white balls, and two black balls. Three balls are drawn from the urn at random without replacement. For each red ball drawn, you win \$10, and for each black ball drawn, you lose \$15. Let X represent your net winnings.

   (a) Compute $P(X = 0)$.
   ANS: $P(WWW) = \dfrac{4}{C(9,3)} = \dfrac{4}{84} = \dfrac{1}{21}$

   (b) Compute $P(X < 0)$.
   ANS: $P(BB) + P(BWW) + P(BRW) = \dfrac{7 + 2 \cdot 6 + 2 \cdot 3 \cdot 4}{84} = \dfrac{43}{84}$

   (c) Compute E(X), your expected net winnings.
   $-30 \cdot P(BBW) - 20P(BBR) - 15P(BWW) - 5P(BRW) + 5P(BRR) + 10P(RWW) + 20P(RRW) + 30P(RRR)$
   $= -30 \cdot \dfrac{4}{84} - 20\dfrac{3}{84} - 15 \cdot \dfrac{12}{84} - 5 \cdot \dfrac{24}{84} + 5 \cdot \dfrac{6}{84} + 10 \cdot \dfrac{18}{84} + 20 \cdot \dfrac{12}{84} + 30\dfrac{1}{84}$
   $= -\dfrac{10}{7} - \dfrac{5}{7} - \dfrac{15}{7} - \dfrac{10}{7} + \dfrac{5}{14} + \dfrac{15}{7} + \dfrac{20}{7} + \dfrac{5}{14} =$ \$0 Should that have been more obvious?

2. (4.4 # 24) To save time, Nivaldo's chemistry professor grades only two randomly chosen problems of a 10-problem assignment. Suppose that Nivaldo has 7 of 10 problems correct on the assignment.

   (a) Compute the probability that both randomly chosen problems are correct.
   ANS: You can compute this in at least two obvious ways: $\dfrac{C(7,2)}{C(10,2)} = \dfrac{21}{45} = \dfrac{7}{15} = \dfrac{7}{10} \cdot \dfrac{6}{9}$

   (b) Compute the probability that both randomly chosen problems are incorrect.
   ANS: $\dfrac{C(3,2)}{C(10,2)} = \dfrac{3}{45} = \dfrac{1}{15} = \dfrac{3}{10} \cdot \dfrac{2}{9}$

   (c) Compute the expected value of the number of correct problems chosen by the professor.
   ANS: $\sum_{n=0}^{10} n \cdot P(n) = 1 \cdot \dfrac{21}{45} + 2 \cdot \dfrac{7}{15} = \dfrac{7}{5}$

3. (4.4 # 26) Conduct the random experiment of flipping a coin five times. Let X be the number of heads.

   (a) Compute $P(X > 3)$.
   $P(X = 4) + P(X = 5) = 5 \cdot \dfrac{1}{2^5} + \dfrac{1}{2^5} = \dfrac{3}{16}$

   (b) Compute $E(X)$.
   $\sum_{n=0}^{5} n \cdot P(n) = (0 \cdot 1 + 1 \cdot 5 + 2 \cdot 10 + 3 \cdot 10 + 4 \cdot 5 + 5 \cdot 1) \cdot \dfrac{1}{32} = \dfrac{80}{32} = 2.5$