Math 15 - Spring 2017 - Homework 4.2 Solutions

1. (4.2 # 18) A certain brand of jellybean comes in four colors: red, green, purple, and yellow. These jellybeans are packaged in bags of 50, but there is no guarantee as to how the colors will be distributed; you might get a mixture of all four colors, or just some red and some green, or even (if you are very lucky) a whole bag of purple.

   (a) Explain how to view the color distribution of a bag of jellybeans as a solution to an equation like the one in Example 4.21.
   ANS: We can think of this as the number of ways we can choose where to place 3 0's amongst 50 1's. The 0's represent the partitioning of the groups of red, green, purple and yellow jellybeans. Say,

   $$11110111111111111111111111011111111110111111111111111$$

   would be an arrangement describing 4 red, 21 green, 10 purple and 15 yellow jellybeans. We want, then, to compute the number of ways of choosing which 3 of 53 symbols are 0's, while the rest are 1's, which is combinations of 53 choose 3, or $\binom{53}{3} = \frac{53 \cdot 52 \cdot 51}{3!} = 46852$ distributions.

   (b) Compute the total number of different possible color distributions.
   I did that.

2. (4.2 # 20) How many different ways are there to distribute 12 identical bones among three different dogs?
   ANS: $\binom{14}{2} = \frac{14 \cdot 13}{2} = 91$

3. (4.2 # 24) Let $ngeq0$ and let j and k be non-negative integers such that $j + k = n$. Use algebra to prove that $C(n, j) = C(n, k)$.
   ANS: $C(n, j) = \frac{n!}{j!(n-j)!} = \frac{n!}{(n-k)!(n-(n-k))!} = C(n, k)$

4. (4.2 # 28) Compute the coefficient of z^4 in the expansion of $(2z + 5)^7$
   ANS: $C(7, 4)(2z)^4 \cdot 5^3 = 35 \cdot 16 \cdot 125z^4 = 7000z^4$