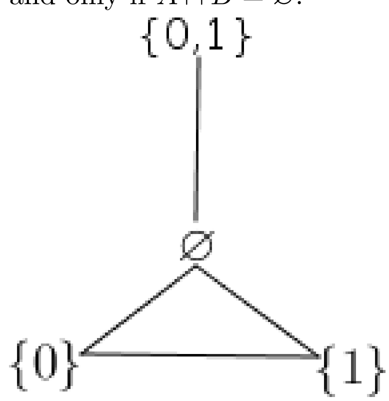
Math 15 - Spring 2017 - Homework 2.2 Solutions

1. (2.2 # 26) Let E be the set of all even integers, and let O be the set of all odd integers. Let $X = \{n \in \mathbb{Z} | n = x + y \text{ for some } x, y \in O\}$.
   (a) Prove that $X \subseteq E$.
   Proof: We want to show that $x \in X \Rightarrow x \in E$. We can do this directly with definitions. Suppose $x \in X$, then $x = (2i+1)+(2j+1)$ for some $i, j \in \mathbb{Z}$. Now using commutativity and associativity of addition on $\mathbb{Z}$, we can rewrite $x = 2i+2j+2$, whence, by the distributive property, $x = 2(i+j+1)$ and since $\mathbb{Z}$ is closed under addition, x is twice an integer and so, by definition, x is even.
   (b) Prove that $E \subseteq X$.
   Proof: We want to show that $e \in E \Rightarrow e \in X$. If $e \in E$ then $e = 2i$ for some $i \in \mathbb{Z}$. But for every $i \in \mathbb{Z}$ there is some $j \in \mathbb{Z}$ such that $i = j + 3$, and, subsituting, $e = 2(j + 3) = 2(j + 2 + 1) = 2j + 4 + 2 = (2j + 1) + 5$, a sum of odd numbers. So $e \in X$.
2. (2.2 # 28) Let A and B be sets. Prove that $A \cap B \subseteq A \cup B$.
   Proof: Let $x \in A \cap B$. Then $(x \in A) \wedge (x \in B) \Rightarrow (x \in A)$ by simplification. Also $(x \in A) \vee (x \in B)$ by addition, so $x \in A \cup B$
3. (2.2 # 30) Draw an undirected graph G with the following properties. The vertices of G correspond to the elements of $P(\{0, 1\})$. To vertices (corresponding to $A, B \in P(\{0, 1\})$) are connected by an edge if and only if $A \cap B = \varnothing$.



4. (2.2 # 32) Let X be any finite set. Consider the graph G described in Exercise 30, replacing $P(\{0, 1\})$ with $P(X)$. Explain why G must be a connected graph.
   ANS: The null set $\varnothing$ is connected to every other vertex in the graph.