Math 15 - Spring 2017 - Homework 1.3 and 1.4

1. (1.3#12) Write the following statements in predicate logic. Define what your predicates are. Use the domain of all quadrilaterals.
   (a) All rhombuses are parallelograms.
   (b) Some parallelograms are not rhombuses.

2. (1.3#14) In the domain of all people, consider the following predicate.

$$P(x, y) = \text{“x needs to love y”}$$

   (a) Write the statement “Everybody needs somebody to love” in predicate logic.
   (b) Formally negate your statement from part (a).
   (c) Write the English translation of your negated statement.

3. (1.3#15) The domain for this problem is some unspecified collection of numbers. Consider the predicate

$$P(x, y) = \text{“}x \text{ is greater than } y$$

   (a) Translate the following statement into predicate logic: “Every number has a number that is greater than it.”
   (b) Negate your expression from part (a), and simplify it so that no quantifier lies within the scope of a negation.
   (c) Translate your expression from part (b) into understandable English. Don’t use variables in your English translation.

4. (1.3#16) Any equation or inequality with variables in it is a predicate in the domain of real numbers. For each of the following statements, tell whether the statement is true or false.
   (a) $\forall x)(x^2 > x)$
   (b) $(\exists x)(x^2 - 2 = 1)$
   (c) $(\exists x)(x^2 - 2 = 1)$
   (d) $(\forall x)(\exists y)(x^2 - y = 4)$
   (e) $(\exists y)(\forall x)(x^2 + y = 4)$

5. (1.3#18)(a) Write the following statement in predicate logic, and negate it. Say what your predicates are, along with the domains.
   Let x and y be real numbers. If x is rational and y is irrational, then $x + y$ is irrational.

6. (1.3#20) Let the following predicates be given in the domain of all triangles.

   $R(x) =$“x is a right triangle.”
   $B(x) =$“x has an obtuse angle.”

   Consider the following statements:

   $S_1 = \neg(\exists x)(R(x) \wedge B(x))$
   $S_2 = (\forall x)(R(x) \rightarrow \neg B(x))$

   (a) Write a proof sequence to show that $S_1 \Leftrightarrow S_2$
   (b) Write S_1 in ordinary English.
   (c) Write S_2 in ordinary English.

7. (1.3#22) Let the following predicates be given. The domain is all cars.

   $F(x) =$ "x is fast."
   $S(x) =$ "x is a sports car."
   $E(x) =$ "x is expensive."
   $A(x, y) =$ "x is safer than y."

   (a) Write the following statements in predicate logic.
      (a) All sports cars are fast.
      (b) There are fast cars that aren't sports cars.
      (c) Every fast sports car is expensive.

   (d) Write the following predicate logic statement in everyday English. Don't just give a word-for-word translation; your sentence should make sense.

   $$(\forall x)[S(x) \rightarrow (\exists y)(E(y) \wedge A(y, x))]$$

   (e) Formally negate the statement from part (b). Simplify your negation so that no quantifier or connective lies within the scope of a negation. State which derivation rules you are using.

   (f) Give a translation of your negated statement in everyday English.

8. (1.3#24)

   (a) Give an example of a pair of predicates $P(x)$ and $Q(x)$ in some domain to show that the $\exists$ quantifier does not distribute over the $\wedge$ connective. THat is, give an example to show that the statements

   $$(\exists x)(P(x) \wedge Q(x) \text{ and } (\exists x)P(x) \wedge (\exists x)Q(x)$$

   are not logically equivalent.

   (b) It is true, however, that $\exists$ distributes over $\vee$. That is,

   $$(\exists x)P(x) \vee Q(x)) \Leftrightarrow (\exists x)(P(x) \vee (\exists x)Q(x)$$

   is an equivalence rule for predicate logic. Verify that your example from part (a) satisfies this equivalence.