Math 15 - Spring 2017 - Homework 1.1 and 1.2

1. (1.1#1) Let the following statements be given.

$p =$"There is water in the cylinders."
$q =$"The head gasket is blown."
$r =$"The car will start."

   (a) Translate the following statement into symbols of formal logic.
   If the head gasket is blown and there's water in the cylinders, the the car won't start.

   (b) Translate the following formal statement into everyday English.
   $r \to \neg(q \vee p)$

2. (1.1#2) Let the following statements be given.

$p =$"You are in Seoul."
$q =$"You are in Kwangju."
$r =$"You are in South Korea."

   (a) Translate the following statement into symbols of formal logic.
   If you are not in South Korea, then you are not in Seoul or Kwangju.

   (b) Translate the following formal statement into everyday English.
   $q \to (r \wedge \neg p)$

3. (1.1#4)Let s be the following statement.
   If you are studying hard, then you are staying up late at night.

   (a) Give the converse of s.

   (b) Give the contrapositive of s.

4. (1.1#6) Give an example of a quadrilateral that shows that the *converse* of the following statement is false.

   If a quadrilateral has a pair of parallel sides, then it has a pair of supplementary angles.

5. (1.1#8) Give an example of a true if-then statement whose converse is also true.

6. (1.1#10) Use truth tables to establish the following equivalencies.

   (a) Show that $\neg(p \vee q)$ is logically equivalent to $\neg p \wedge \neg q$.

   (b) Show that $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$.

   These equivalencies are known as *De Morgan's laws*, after the nineteenth century logician Augustus De Morgan.

7. (1.1#12) Use truth tables to show that $(a \vee b) \wedge (\neg(a \wedge b))$ is logically equivalent to $a \leftrightarrow \neg b$. (This arrangement of T/F values is sometiemes called the *exclusive or* of a and b.)

8. (1.1#14) Let the following statements be given.

$p =$"Andy is hungry."
$q =$"The refrigerator is empty."
$r =$"Andy is mad."

   (a) Use connectives to translate the following statement into formal logic.

   If Andy is is hungry and the refrigeartor is empty, then Andy is mad.

   (b) Construct a truth table for the statement in part (a).

   (c) Suppose that the statement given in part (a) is true, and suppose also that Andy is not mad and the regrigerator is empty. Is Any hungry? Explain how to justify your answer using the truth table.

9. (1.1#16) Use truth tables to prive the following *distributive properties* for propositional logic.

   (a) $p \wedge (q]veer)$ is logically equivalent to $(p \wedge q) \vee (p \wedge r)$

(b) $p \vee (q \wedge r)$ is logically equivalent to $(p \vee q) \wedge (p \vee r)$

10. (1.1#18) Mathematicians say that "statement P is *stronger* than statement Q" if Q is true whenever P is true, but not conversely. (in other words, "P is stringer that Q" means that $P \to Q$ is always true, but $Q \to P$ is not true, in general.) Use truth tables to show the following.
    (a) $a \wedge b$ is stronger that a.
    (b) a is stronger than $a \vee b$
    (c) $a \wedge b$ is stronger than $a \vee b$
    (d) b is stronger that $a \to b$.
11. (1.1#20) Which statement is stronger?
    - Q is a square.
    - Q is a rectangle.

    Explain
12. (1.1#22) Mathematicians say that "Statement P is a sufficient condition for statement Q" if $P \Rightarrow Q$ is true. In other words, in order to know that Q is true, it is sufficient to know that P is true. Let x be an integer. Give a sufficient condition on x for $x/2$ to be an integer.
13. (1.1#24) : Let Q be a quadrilateral. Give a sufficient but not necessary condition for Q to be a parallelogram.
14. (1.1#26) Often a complicated expression in formal logic can be simplified. For example, consider the statement $S = (p \wedge q) \vee (p \wedge \neg q)$.
    (a) Construct a truth table for S.
    (b) Find a simpler expression that is logically equivalent to S.
15. (1.1#28) The NAND connective $\uparrow$ is defined by the following truth table.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

    Use truth tables to show that $p \uparrow q$ is logically equivalent to $\neg(p \wedge q)$. (This explains the name NAND: Not AND.)
16. (1.1#30) Write $\neg p$ in terms of p and $\uparrow$
17. Use symbols of propositional logic to explain the difference between the following two statements.

$p =$"You are in Seoul."
$q =$"My team will win if I yell at the TV."
$r =$"My team will win only if I yell at the TV."

18. (1.2#2) Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statements	Reasons
1. $q \wedge r$	given
2. $\neg(\neg p \wedge q)$	given
3. $\neg\neg p \vee \neg q$	
4. $p \vee \neg q$	
5. $\neg q \vee p$	
6. $q \to p$	
7. q	
8. p	

19. (1.2#6) Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statements	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	
4. $\neg q \vee \neg p$	
5. $q \to \neg p$	
6. p	
7. $\neg\neg p$	
8. $\neg q$	
9. $(q \vee r) \wedge p$	
10. $q \vee r$	
11. $r \vee q$	
12. $\neg(\neg r) \vee q$	
13. $\neg r \to q$	
14. $\neg(\neg r)$	
15. r	
16. $p \wedge r$	

20. (1.2#8) Which derivation rule justifies the following argument?

    If n is a multiple of 4, then n is even. However, n is not even. Therefore, n is not a multiple of 4.

21. (1.2#10) Let Q be a quadrilateral. Given the statements

    If Q is a rhombus, then Q is a parallelogram.
    Q is not a parallelogram.

    what statement follows by *modul tollens*?

22. (1.2#12) Write a statement that follows from the statement

    It is sunny and warm today.

    by the simplification rule.

23. (1.2#14) Recall Exercise 31 of Section 1.1. Suppose that all the following status reports are correct:

    - Processor B is not working and processor C is working.
    - Processor A is working if and only if processor B is working.
    - At least one of the two processors A and B is not working.

Let $a =$"A is working," $b =$"B is working," and $c =$"C is working."

(a) If you haven't already done so, write each status report in terms of a, b, and c, using the symbols of formal logic.

(b) How would you justify the conclusion that B is not working? (In other words, given the statements in part (a), which derivation rule allows you to conclude $\neg b$?)

(c) How would you justify the conclusion that C is working?

(d) Write a proof sequence to conclude that A is not working. (In other words, given the statements in part (a), write a proof sequence to conclude $\neg a$.)

24. (1.2#16) Write a proof sequence for the following assertion. Justify each step.

$$\left.\begin{array}{r} p \\ p \to r \\ q \to \neg r \end{array}\right\} \Rightarrow \neg q$$

25. (1.2#18) Write a proof sequence for the following assertion. Justify one of the steps in your proof using the result of Example 1.8.

$$\left.\begin{array}{r} \neg(a \wedge \neg b) \\ \neg b \end{array}\right\} \Rightarrow \neg\neg a$$

26. (1.2#20) Write a proof sequence to establish that $p \Leftrightarrow p \wedge p$ is a tautology.

27. (1.2#22) Write a proof sequence for the following assertion. Justify each step.

$$(p \vee q) \vee (p \vee r) \Rightarrow \neg r \to (p \vee q)$$

28. (1.2#26) This exercise will lead you through a proof of the distributive property of $\wedge$ over $\vee$. We will prove:

$$p \wedge (q \vee r) \Rightarrow (p \wedge q) \vee (p \wedge r)$$

(a) The above assertion is the same as the following:

$$p \wedge (q \vee r) \Rightarrow \neg(p \wedge q) \to (p \wedge r)$$

Why?

(b) Use the deduction method from Exercise 25(x) to rewrite the tautology from part (a).

(c) Is $a \to \neg a$ a contradiction? Why or why not?