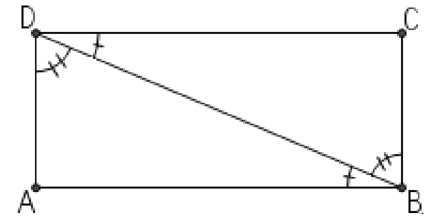
Math 5 – Trigonometry – Fair Game for Chapter 1 Test

Show all work for credit. Write all responses on separate paper.

12. What angle has the same measure as its complement? How do you know?

12. What is the complement of the supplement of 100˚ angle?

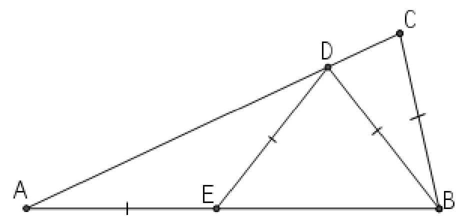
12. Given Quadrilateral *ABCD* with diagonal *BD* forming congruent angles $\angle CDB \cong \angle ABD$ and $\angle CBD \cong \angle ADB$, what kind of quadrilateral do you think this is? Why? Give as persuasive a justification as you can.



12. Find the perimeter of an isosceles triangle with base = 12cm and height = 4cm.

12. Find the area of an isosceles triangle with sides of length 2cm, 3cm and 3cm.

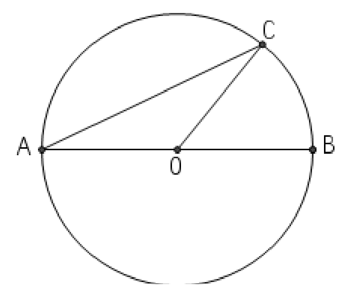
12. Given that $AC = AB$ and $AE = ED = DB = BC$ in ΔABC at right, find the degree measure of $\angle BAC$



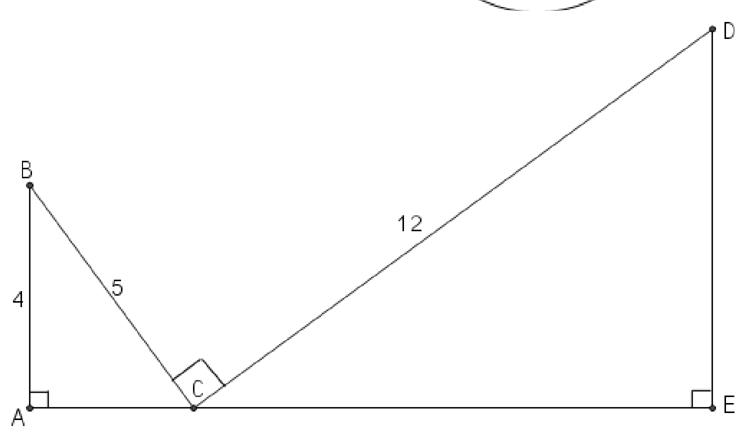
12. Carlos and Karla start at the north west corner of a square block measuring 120 meters on a side and Carlos starts walking around the block by heading east at 0.5 meters per second at the same time that Karla starts walking around the block by heading south at 0.6 meters per second. What is the length of the line segment connecting Carlos and Karla after five minutes?

12. Consider a 120˚ sector of a circle with radius 10cm.
    a. Find the perimeter of the sector.
    b. Find the area of the sector.

12. In the diagram at right, *AB* is a diameter of the circle centered at O and *C* is a point on the perimeter of the circle.
    a. Express $\angle BOC$ in terms of $\angle AOC$.
    b. Explain why ΔAOC is isosceles.
    c. Express $\angle BAC$ in terms of $\angle AOC$.
    d. Express $\angle BAC$ in terms of $\angle BOC$.

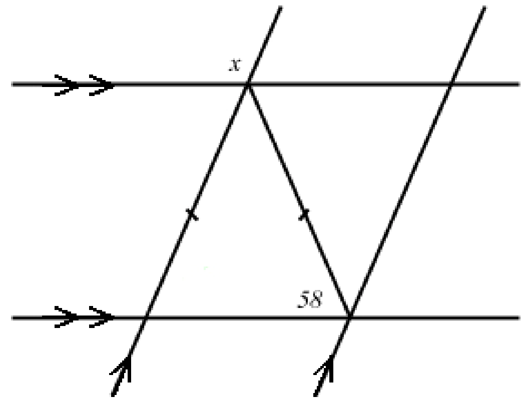


12. In the figure at right,
    a. Show that $\Delta ABC \sim \Delta EDC$
    b. Find the length of AE.
    c. Draw *BD* and find its length. Is $\Delta DCB \sim \Delta ABC$? Why or why not?



12. Explain why the two acute angles of a right triangle are complementary.

12. What is the degree measure of angle x in the figure at right? Explain how you know.

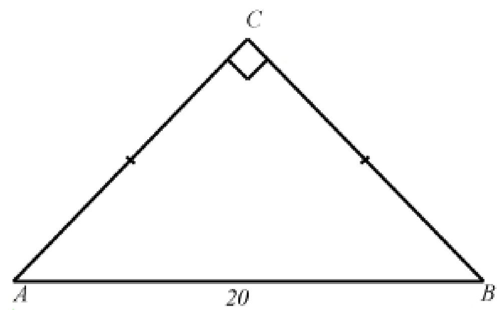


12. True or false: if one side of a quadrilateral is congruent to the opposite side, then the quadrilateral is either a parallelogram or an isosceles trapezoid. Justify your answer.

12. In the isosceles right triangle shown at right, AB = 20.

(a) What is the length of AC?

(b) Draw the altitude from C to AB. What is its length?



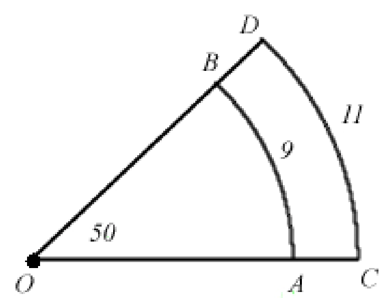
12. Two boats leave a dock at the same time and at a 90^{o} angle from each other. After 3 hours one boat is 10 miles from the dock, while the other is 40 miles from the dock. How far are the boats from each other? Write your answer in simplest radical form.

12. Find the (a) perimeter and (b) area of a rectangle with one side 10 cm and diagonal 13 cm.

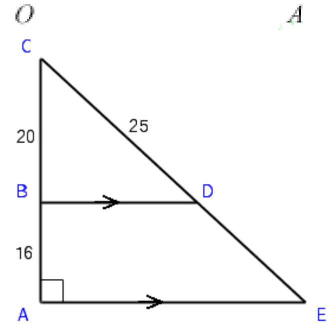
12. Explain why a diagonal of a parallelogram creates two congruent triangles.

12. Find the area of region ACDB. $AB = 9$cm and $CD = 11$cm are concentric arcs with center O .

12. A closed right circular cylinder has a radius of 3 meters. Find the volume of the cylinder if its lateral surface area is 84π square meters. Leave your answer in terms of π.



12. Show that triangles BCD and ACE are similar and use that similarity to find the length of DE in the diagram at right.



Math 5 Chapter 1 Review Solutions

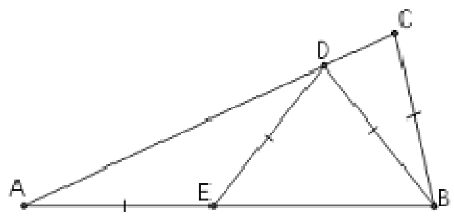
1. If x is its own complement then $x + x = 90^\circ$, or $x = 45^\circ$.

2. The supplement of 100° is 80° whose complement is 10°.

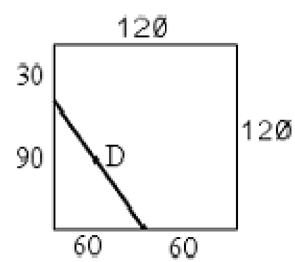
3. We know that if a transversal cuts two parallel lines, alternate interior angles are congruent. It seems reasonable that the converse is also true: if the alternate interior angles are congruent, then the lines are parallel. Thus AB and CD are parallel and AD and BC are parallel, so quadrilateral ABCD is a parallelogram.

4. An isosceles triangle with base = 12cm and height = 4cm has equal sides of length $\sqrt{4^2+6^2}=\sqrt{52}=2\sqrt{13}$ Thus the perimeter is $12+4\sqrt{2}\text{cm}$.

5. The height of an isosceles triangle with sides of length 2cm, 3cm and 3cm is $\sqrt{3^2-1^2}=\sqrt{8}=2\sqrt{2}$, so its area is $2\sqrt{2}$ cm^2.

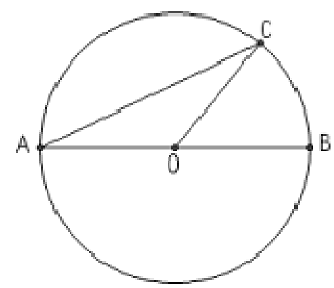


6. Let $x = \angle CAB$ then, since ΔAED is isosceles with base AD, $\angle ADE = x$ and since the sum of interior angles is 180°, $\angle AED = 180^\circ - 2x^\circ$ Further, since $\angle AEB$ is a straight angle, $\angle DEB = 2x$ is the supplement of $\angle AED$. But ΔEDB is isosceles, so $\angle DBE - \angle DEB - 2x^\circ$ and thus (interior angles' sum is 180°) $\angle EDD - 180^\circ - 4x^\circ$. Now since $\angle ADC \angle ADC$ is a straight angle, $\angle ADC + \angle EDB + \angle BDC = 180^\circ$ and, substituting, $x+(180-4x)+\angle BDC = 180$ whence $\angle BDC = 3x^\circ$ Also, ΔCBD is isosceles so $\angle BCD = \angle BDC = 3x^\circ$. Finally, since ΔBAC is isosceles, $\angle BCD = (180-x)/2$ so we have the equation $3x = (180-x)/2 \Leftrightarrow \boxed{x^\circ = 180^\circ/7}$



7. Since 5 minutes is 300 seconds, Carlos will travel (0.5 m/s)*300s = 150m, putting him at a point 30 m south of the north edge on the east side of the block. Karla will travel (0.6 m/s)*300s = 180m, so that she's on the south edge, 60m east of the west side. Thus the distance between them is the hypotenuse of a right triangle with legs 60 and 90, $D=\sqrt{60^2+90^2}=\sqrt{3600+8100}=30\sqrt{13}\approx 108.2\,\text{m}$

8. Consider a 120° sector of a circle with radius 10cm.

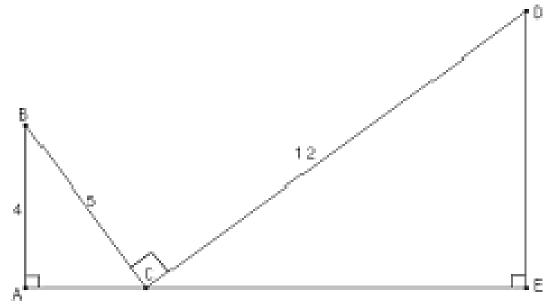


   a. The perimeter of the sector is

$$2(10)+\frac{120}{360}(20\pi)=20+\frac{20\pi}{3}$$

   b. The area of the sector is $\frac{120}{360}\pi(10)^2=\frac{100\pi}{3}\text{cm}^2$

9. In the diagram at right, AB is a diameter of the circle centered at O and C is a point on the perimeter of the circle



a. $\angle DOC - 180^\circ - \angle AOC \angle BOC = 180^\circ - \angle AOC$.

b. AD and DC are radii, so $AD = DC$.

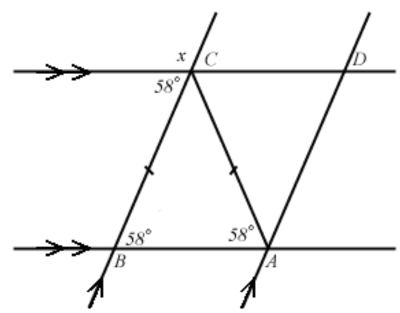
c. $\angle BAC = \dfrac{180^\circ - \angle AOC}{2}$

d. $\angle BOC = 2\angle BAC \angle BOC = 2\angle BAC$

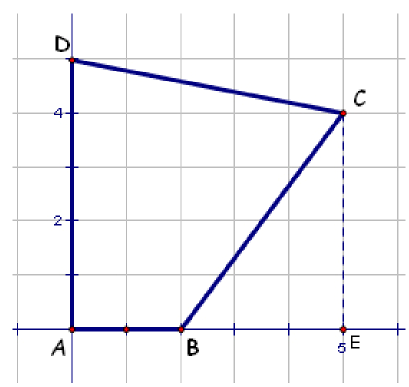
10. $\angle BCA + \angle ABC = \angle BCA + \angle DCE$ so $\angle ABC = \angle DCE$ and thus $\Delta ABC \sim \Delta EDC$.
So CE/12 = 4/5 and CE = 9.6 whence AE = 3 + 9.6 = 12.6.
BD = 13 and 13/12 is not equal to 5/4 so ΔABC is not similar to ΔBDC

11. Explain why the two acute angles of a right triangle are complementary.
ANS: The sum of the interior angles of any triangle is 180°. In a right triangle, one angle is 90° and that means the remaining angles add up to 90°, which means they're complementary.

12. What is the degree measure of angle *x* in the figure at right? Explain how you know
ANS: Since ΔABC is isosceles, $\angle ABC$ = 58°, and since $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, *x* must be supplementary to the corresponding angle at *C* and thus *x* = 122°.



13. True or false: if one side of a quadrilateral is congruent to the opposite side, then the quadrilateral is either a parallelogram or an isosceles trapezoid. Justify your answer.
ANS: False. Consider the quadrilateral *ABCD* with vertices at *A*(0,0), *B*(2,0), *C*(5,4) and *D*(0,5), as shown at right.
In the figure, *AD* = *BC* = 5, but the figure is neither a parallelogram nor an isosceles trapezoid.



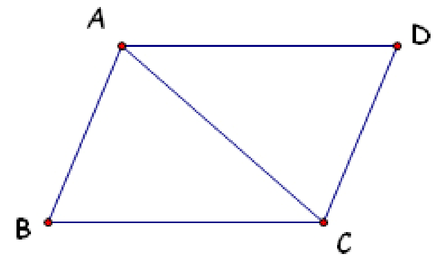
14. In the isosceles right triangle shown at right, AB = 20.
(a) What is the length of AC?
ANS: AC/AB = AC/20 = $\sqrt{2}/2$ so $AC = 10\sqrt{2}$

(b) Draw the altitude from C to AB. What is its length?
NS: This line would create two congruent isosceles right triangles each half the size of the original. Thus the altitude would be 10.

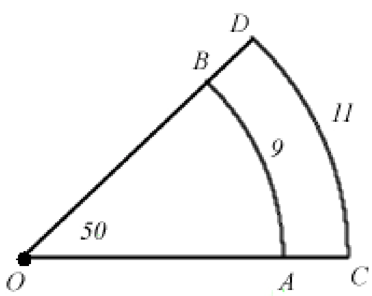
15. Two boats leave a dock at the same time and at a 90° angle from each other. After 3 hours one boat is 10 miles from the dock, while the other is 40 miles from the dock. How far are the boats from each other? Write your answer in simplest radical form.
ANS: The boats' paths are legs of a right triangle and the distance between them is the hypotenuse of that triangle: $D = \sqrt{10^2 + 40^2} = \sqrt{100 + 1600} = \sqrt{1700} = 10\sqrt{17}$

16. Find the (a) perimeter and (b) area of a rectangle with one side 10 cm and diagonal 13 cm.
ANS: The short side of the rectangle is $\sqrt{13^2-10^2}=\sqrt{169-100}=\sqrt{69}$ so the perimeter is $2(10)+2\sqrt{69}=20+2\sqrt{69}$ and the area is $10\sqrt{69}$

17. Explain why a diagonal of a parallelogram creates two congruent triangles
ANS: The transversal AC forms alternate interior angles congruent so that $\angle CAB \cong \angle ACD$ and $\angle CAD \cong \angle ACB$. Also, AC is congruent to itself. This establishes the conditions of the ASA so we can conclude the triangles are congruent.



18. Find the area of region $ACDB$. AB = 9cm and CD = 11cm are concentric arcs with center O.
ANS: Solve the arc length formula $s=\frac{\angle A}{180}\pi r$ for the radius and plug in the values of s:

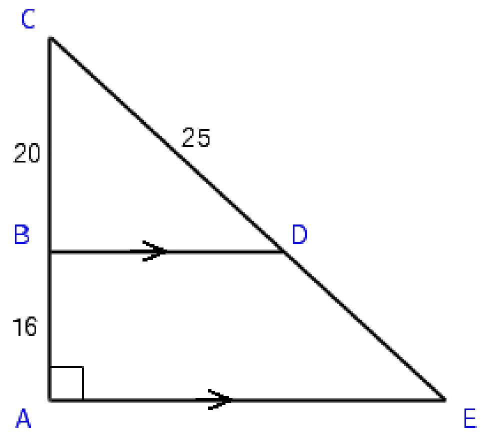


$r_1=\frac{180}{\pi\angle A}s=\frac{180}{50\pi}(9)=\frac{162}{5\pi}$ and $r_2=\frac{180}{50\pi}(11)=\frac{198}{5\pi}$ and then use these in the formula $A=\frac{50}{360}\left(\pi r^2\right)$ for the area of a sector to compute the difference of the sectors' areas:

$$\frac{5\pi}{36}\left(\frac{6(33)}{5\pi}\right)^2-\frac{5\pi}{36}\left(\frac{6(27)}{5\pi}\right)^2=\frac{33^2}{5\pi}-\frac{27^2}{5\pi}=\frac{(33-27)(33+27)}{5\pi}=\frac{72}{\pi}\text{ cm}^2.$$

19. A closed right circular cylinder has a radius of 3 meters. Find the volume of the cylinder if its lateral surface area is 84π square meters. Leave your answer in terms of π.
ANS: The formula for lateral surface area is $2\pi rh$. Substituting $r=3$ and setting this equal to 84π we have $6\pi h=84\pi$ whence $h=14$, thus the volume is $V=\pi r^2h=\pi 9(14)=126\pi$.

20. Show that triangles BCD and ACE are similar and use that similarity to find the length of DE in the diagram.



ANS: Since a transversal cutting parallel lines makes corresponding angles congruent, $\angle CBD$ is a right angle. Also triangles *BCD* and *ACE* share an angle at *A*. Since two of the angles are equal, and the sum of interior angles is 180°, it must be that the third angles are also equal and thus the triangles are equiangular, which also means they are similar. Let $x = ED$. Then, since corresponding parts of similar triangles are proportional, $\frac{x+25}{25} = \frac{36}{20} \Leftrightarrow x = 20$.