Math 5 – Trigonometry – fall ’12 – Chapter 2 Test   Name____________________________

Show all work for credit. Write all responses on separate paper. Do not use a calculator.

1. Find the domain of each of the following functions.
   a. $f(x) = \sqrt{8 - x}$
   b. $g(x) = \frac{1}{8 - x^2}$

2. Consider the line passing through the origin (0,0) and a point on the parabola described by $f(x) = (x - 4)^2 + 2$
   a. Using the point $(h, f(h))$ on the function and the point (0,0) at the origin, show that a formula for the function $m(h)$ that describes the slope of this line as a function of h is $m(h) = h - 8 + \frac{18}{h}$.
   b. Evaluate and simplify $m(3)$, $m(4)$ and $m(5)$.

3. Compute and simplify the average rate of change of $f(x) = 2x^3$ over the given interval. Hint: recall that the average rate of change is of $y = f(x)$ on the interval $[a, b]$ is the slope of the secant line connecting $(a, f(a))$ with $(b, f(b))$.
   a. $[0, h]$
   b. $[-h, h]$

4. Consider the quadratic $f(x) = 3x^2 - 6x + 2$
   a. Express the quadratic function in standard (vertex) form: $y = a(x - h)^2 + k$
   b. Find the coordinates of the x-intercepts.
   c. Express the quadratic function in factored form: $y = a(x - r_1)(x - r_2)$
   d. Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.

5. Find the range of the given function and express that in interval notation.
   a. $f(x) = -4(x-1)^2 + 10$
   b. $f(x) = -2x^2 + 8x + 1$

6. Consider the quadratic $f(x) = 5 - \frac{1}{2}(x + 3)^2$. What sequence of vertical shift, reflection, vertical stretch, and horizontal shift is required to transform this function to $y = x^2$?

7. Suppose $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x - 2}$.

   Find a formula for and determine the domain of $(g \circ f)(x)$

8. Find a formula for the inverse function of $f(x) = \frac{1}{2}x + 1$ and sketch a graph for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y = x$.

Math 5 – Trigonometry – fall '12 – Chapter 2 Test Solutions

1. Find the domain of each of the following functions.
   a. $f(x) = \sqrt{8-x}$
   ANS: The domain is $(-\infty, 8]$
   b. $g(x) = \frac{1}{8-x^2}$
   ANS: The domain is $\left(-\infty, -2\sqrt{2}\right) \cup \left(-2\sqrt{2}, 2\sqrt{2}\right) \cup \left(2\sqrt{2}, \infty\right) = \left\{x | x \neq \pm 2\sqrt{2}\right\}$

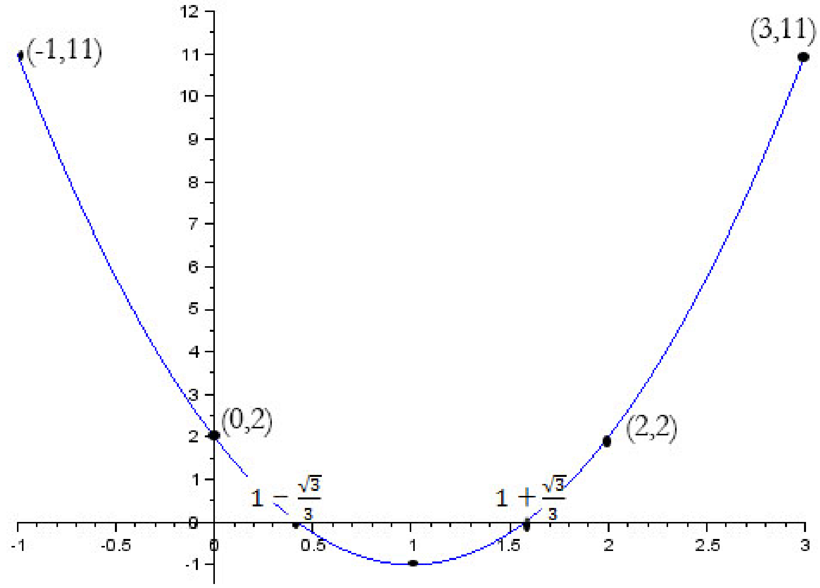
2. Consider the line passing through the origin (0,0) and a point on the parabola described by $f(x) = (x-4)^2 + 2$
   a. Using the point $\left(h, f(h)\right)$ on the function and the point (0,0) at the origin, show that a formula for the function $m(h)$ that describes the slope of this line as a function of h is $m(h) = h - 8 + \frac{18}{h}$.
   ANS: The slope is $\frac{f(h)-0}{h-0} = \frac{(h-4)^2+2}{h} = \frac{h^2-8h+18}{h} = h - 8 + \frac{18}{h}$
   b. Evaluate and simplify $m(3)$, $m(4)$ and $m(5)$.
   ANS: $m(3) = 3 - 8 + 6 = 1$, $m(4) = 4 - 8 + \frac{9}{2} = \frac{1}{2}$, and $m(5) = 5 - 8 + \frac{18}{5} = \frac{3}{5}$
   Do you see what happened there? It went down at first…and then…went back up some!

3. Compute and simplify the average rate of change of $f(x) = 2x^3$ over the given interval. Hint: recall that the average rate of change is of $y = f(x)$ on the interval $[a, b]$ is the slope of the secant line connecting $(a, f(a))$ with $(b, f(b))$.
   a. $[0, h]$
   ANS: $\frac{f(h)-f(0)}{h-0} = \frac{2h^3}{h} = 2h^2$
   b. $[-h, h]$
   ANS: $\frac{f(h)-f(-h)}{h-(-h)} = \frac{2h^3-(-2h^3)}{2h} = 2h^2$ Hey…it's the same!

4. Consider the quadratic $f(x) = 3x^2 - 6x + 2$
   a. Express the quadratic function in standard (vertex) form: $y = a(x-h)^2 + k$
   SOLN: $f(x) = 3(x^2 - 2x) + 2 = 3(x^2 - 2x + 1) + 2 - 3 = 3(x-1)^2 - 1$
   b. Find the coordinates of the x-intercepts.
   SOLN: At the x-intercepts, $f(x) = 0 \Leftrightarrow 3(x-1)^2 = 1 \Leftrightarrow (x-1)^2 = \frac{1}{3} \Leftrightarrow x - 1 = \pm\frac{\sqrt{3}}{3}$
   So the intercepts are where $x = 1 \pm \frac{\sqrt{3}}{3}$
   c. Express the quadratic function in factored form: $y = a(x-r_1)(x-r_2)$

$$y = 3\left(x - 1 + \frac{\sqrt{3}}{3}\right)\left(x - 1 - \frac{\sqrt{3}}{3}\right)$$

   d. Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.



5. Find the range of the given function and express that in interval notation.
   a. $f(x) = -4(x-1)^2 + 10$
      ANS: $(-\infty, 10]$
   b. $f(x) = -2x^2 + 8x + 1$
      ANS: $f(x) = -2x^2 + 8x + 1 = -2(x-2)^2 + 9$ so the range is $(-\infty, 9]$

6. Consider the quadratic $f(x) = 5 - \frac{1}{2}(x+3)^2$. What sequence of vertical shift, reflection, vertical stretch, and horizontal shift is required to transform this function to $y = x^2$?
   ANS: Shift down 5 to get $y = -\frac{1}{2}(x+3)^2$, then reflect across the x-axis to get $y = \frac{1}{2}(x+3)^2$, then stretch vertically by a factor of 2 to get $y = (x+3)^2$. Finally shift 3 to the right to get $y = x^2$.

7. Suppose $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x-2}$.
   Find a formula for and determine the domain of $(g \circ f)(x)$
   ANS: $(g \circ f)(x) = g(\sqrt{x}) = \frac{1}{\sqrt{x}-2}$ has domain $[0,4) \cup (4, \infty)$.

8. Find a formula for the inverse function of $f(x) = \frac{1}{2}x + 1$ and sketch a graph for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y = x$.
   ANS: Following the procedure to find a formula for the inverse function, we write, $y = \frac{1}{2}x + 1$ and solve for x: $\frac{1}{2}x = y - 1 \Leftrightarrow x = 2y - 2$, and then swap variables, $x \leftrightarrow y$ to get

   $f^{-1}(x) = 2x - 2$