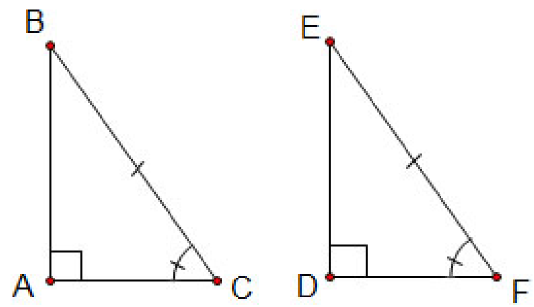
Math 5 – Trigonometry – Geometry Test – Fall ’12 Name________________________________

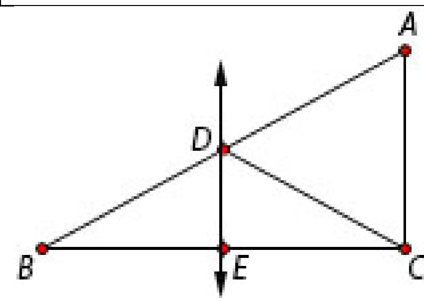
Show your work for credit. Except for problems 1 and 2, write all responses on separate paper. Do not use a calculator.

1. (HA Theorem) Given right triangles ΔABC, ΔDEF with $\angle A$ and $\angle D$ right angles and $\overline{BC} \cong \overline{EF}$ and $\angle C \cong \angle F$; fill in the missing statements to complete the proof that $\Delta ABC \cong \Delta DEF$



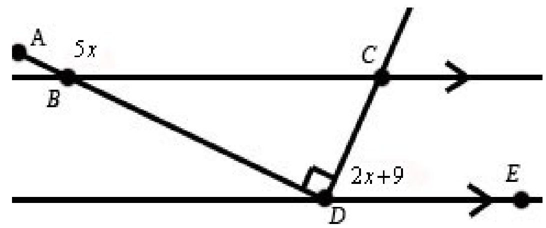
Statement	Reason
	Given
	All right angles are congruent
	Given
	AAS

2. Theorem: The median from the right angle in a right triangle is one-half the length of the hypotenuse.
Given: ΔABC with right angle $\angle ACB$ and median $\overline{CD}$.
Prove: $CD = \frac{1}{2} AB$



Statement	Reason
Draw $\overleftrightarrow{DE} \| \overline{AC}$	
ΔABC with right angle $\angle ACB$ and median $\overline{CD}$	
$BD = AD$	Definition of median and def. of ≅ segments.
$\frac{AD}{DB} = \frac{CE}{EB}$	A line || to one side and intersecting two sides of a triangle dives the sides into proportional segments.
$\overline{DE} \cong \overline{DE}$	
	If two legs of two right Δs are ≅, the Δs are ≅.
$BD = CD$	
$BD + DA = BA$	
$BD + BD = BA$	
$2BD = BA$	
	Substitution postulate
$CD = \frac{1}{2} BA$	

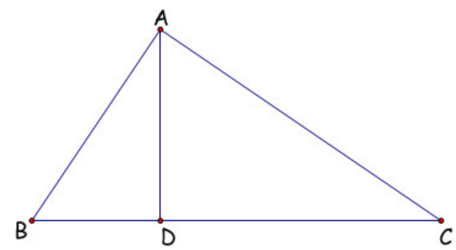
3. In the figure at right, $\overline{BC} \| \overline{DE}$.
   What is the degree measure of x
   so that $\angle ABC = 5x$ and $\angle CDE = 2x + 9$?



4. Find the altitude of an equilateral triangle with sides of length 4.

5. Draw an isosceles right triangle with hypotenuse of length 8 inches.
   Find the perimeter and area of this triangle.

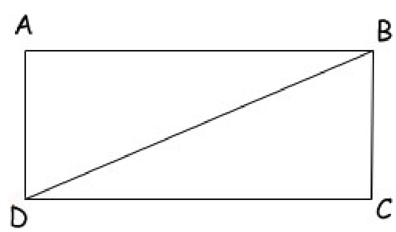
6. Consider the diagram at right and assume that $\overline{AB} \perp \overline{AC}$ and that $\overline{AD} \perp \overline{BC}$.



   a. Prove that $\Delta CAD \sim \Delta ABD$

   b. If $AC = 15$ and $AD = \frac{120}{17}$, find the area of ΔABD. Hint: If you find CD then you'll have the ratios $\frac{\text{hypotenuse}}{\text{short leg}}, \frac{\text{hypotenuse}}{\text{long leg}}, \frac{\text{long leg}}{\text{short leg}}$ for all three triangles

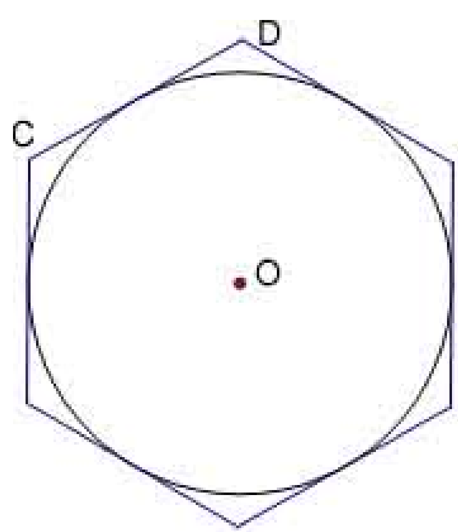
7. Consider quadrilateral $ABCD$ shown at right and suppose we know that $\angle ADB \cong \angle CBD$.
   What type of quadrilateral can we deduce that $ABCD$ is?



8. Consider the circle inscribed in a regular hexagon, as shown in the diagram to the right.

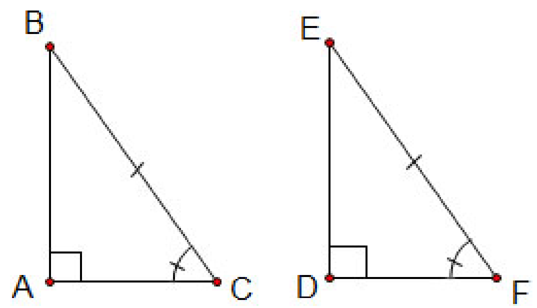
   a. What is the measure of central angle $\angle COD$?

   b. If $CD = 8$, what is the area of the circle?



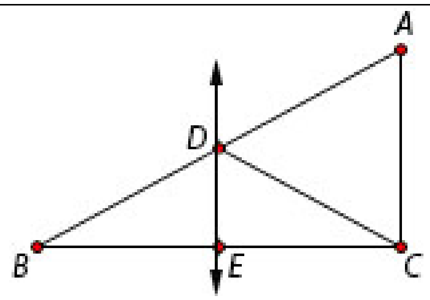
Math 5 – Trigonometry – Geometry Test Solutions – Fall '12

1. (HA Theorem) Given right triangles ΔABC, ΔDEF with $\angle A$ and $\angle D$ right angles; $\overline{BC} \cong \overline{EF}$ and $\angle C \cong \angle F$, fill in the missing statements to complete the proof that $\Delta ABC \cong \Delta DEF$



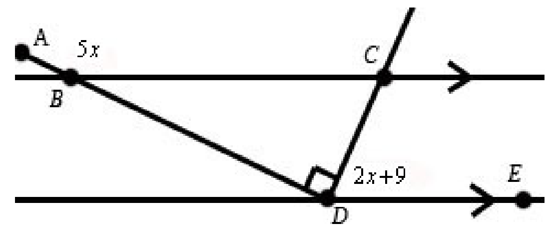
Statement	Reason
right triangles ΔABC, ΔDEF with $\angle A$ and $\angle D$ right angles	Given
$\angle A \cong \angle D$	All right angles are congruent
$\angle C \cong \angle F$ and $\overline{BC} \cong \overline{EF}$	Given
$\Delta ABC \cong \Delta DEF$	AAS

2. Theorem: The median from the right angle in a right triangle is one-half the length of the hypotenuse.
Given: ΔABC with right angle $\angle ACB$ and median $\overline{CD}$.
Prove: *CD* = ½ *AB*



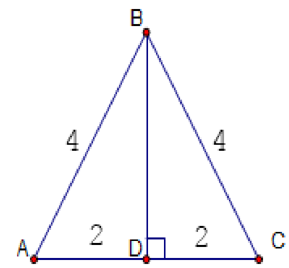
Statement	Reason
Draw $\overleftrightarrow{DE} \| \overline{AC}$	**Parallel postulate**.
ΔABC with right angle $\angle ACB$ and median $\overline{CD}$	**Given**
BD = *AD*	Definition of median and def. of ≅ segments.
$\frac{AD}{DB} = \frac{CE}{EB}$	A line || to one side and intersecting two sides of a triangle dives the sides into proportional segments.
$\overline{DE} \cong \overline{DE}$	**Reflexive Postulate**
$\Delta BDE \cong \Delta CDE$	If two legs of two right Δs are ≅, the Δs are ≅.
BD = *CD*	**CPCTC**
BD + *DA* = *BA*	**Partition Postulate**
BD + *BD* = *BA*	**Substitution Postulate**
2*BD* = *BA*	**Distributive Postulate**
2*CD* = *BA*	Substitution postulate
CD = ½ *BA*	**Division Postulate**

3. In the figure at right, $\overline{BC} || \overline{DE}$.
What is the degree measure of x
so that $\angle ABC = 5x$ and $\angle CDE = 2x + 9$?



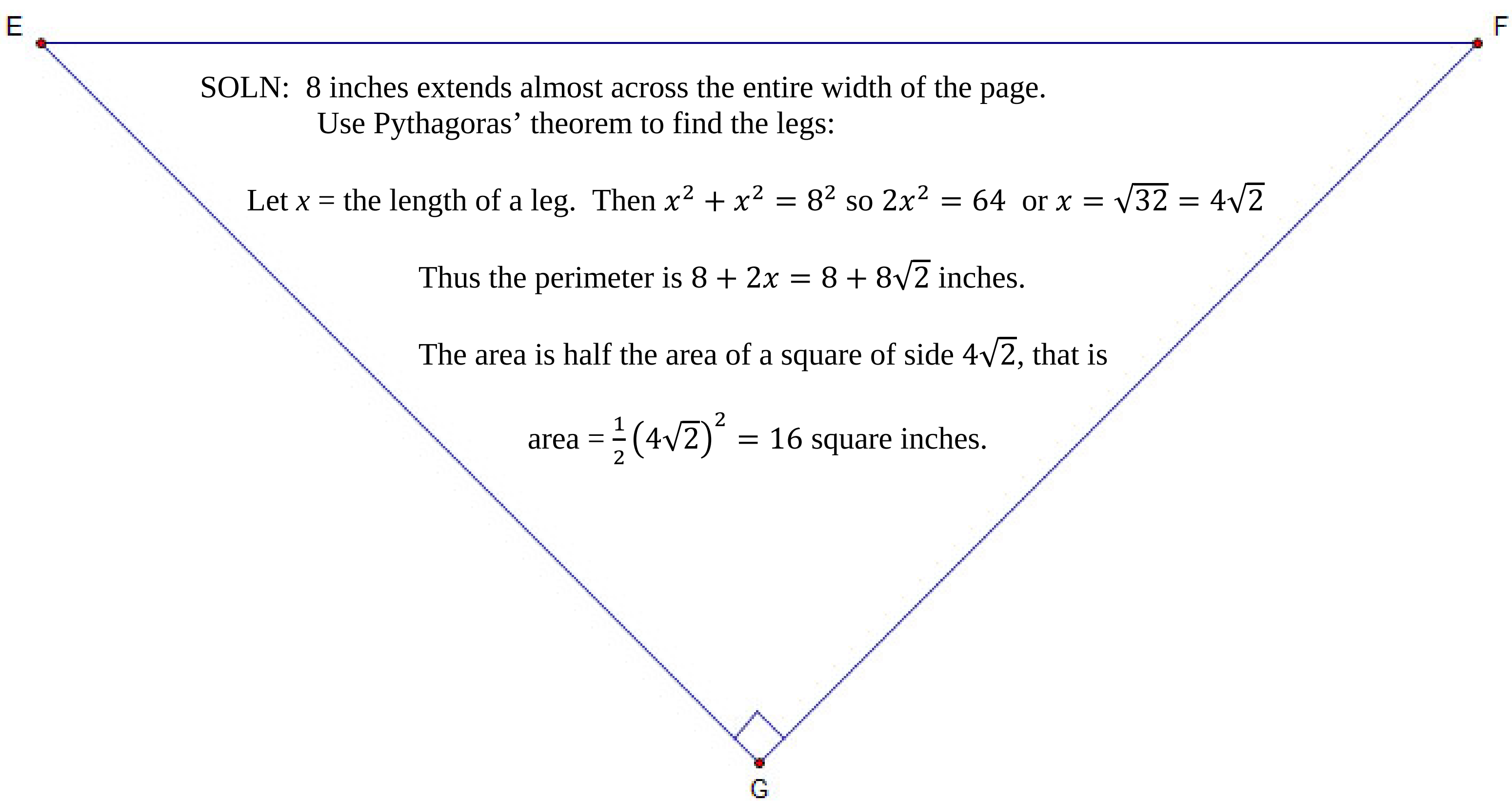
SOLN: $\angle CBD = 180 - 5x$ (*A-B-C* is straight) and $\angle BCD = 2x + 9$ (when *CD* cuts $\overline{BC} || \overline{DE}$, alternate interior angles are congruent). Since the sum of the interior angles of a triangle is 180,
$180 - 5x + 90 + 2x + 9 = 180 \Leftrightarrow 3x = 99$ so $x = 33$. Alternatively, note that when $\overline{BD}$ cuts $\overline{BC} || \overline{DE}$, corresponding angles are congruent, so $\angle ABC = \angle BDE$. Thus $5x = 2x + 99$ and so $x = 33$.

4. Find the altitude of an equilateral triangle with sides of length 4.
SOLN:



Draw $\overline{BD} \perp \overline{AC}$ Then, since base angles $\angle A \cong \angle C$ and $\overline{BD} \cong \overline{BD}$, $\Delta ABD \cong \Delta CBD$ by AAS. Therefore $\overline{AD} \cong \overline{CD}$ (CPCTC) so that *AD* = 2. Now using Pythagoras' theorem, we have the altitude *BD* satisfies $2^2 + BD^2 = 4^2$ so that $BD^2 = 12$ and $BD = 2\sqrt{3}$. As a shortcut, one could observe that the altitude is the longer leg of a 30-60-90 triangle and so its length is $\sqrt{2}$ times the shorter leg.

5. Draw an isosceles right triangle with hypotenuse of length 8 inches.
Find the perimeter and area of this triangle.



SOLN: 8 inches extends almost across the entire width of the page.
Use Pythagoras' theorem to find the legs:

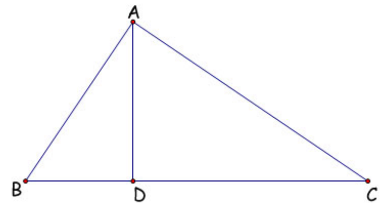
Let x = the length of a leg. Then $x^2 + x^2 = 8^2$ so $2x^2 = 64$ or $x = \sqrt{32} = 4\sqrt{2}$

Thus the perimeter is $8 + 2x = 8 + 8\sqrt{2}$ inches.

The area is half the area of a square of side $4\sqrt{2}$, that is

area $= \frac{1}{2}\left(4\sqrt{2}\right)^2 = 16$ square inches.

6. Consider the diagram at right and where $\overline{AB} \perp \overline{AC}$ and $\overline{AD} \perp \overline{BC}$.



a. Prove that $\Delta CAD \sim \Delta ABD$

Statement	Reason
$\angle ADC \cong \angle ADB \cong \angle CAB$	All rt $\angle s$ are $\cong$
$m\angle ADC = m\angle ADB = m\angle CAB$	$\cong$ $\angle s$ have equal measure
$\angle B + \angle BAD + 90° = 180°$ $\angle B + \angle C + 90° = 180°$ $\angle CAD + \angle C + 90° = 180°$	The sum of interior angles of a triangle is 180°.
$\angle B + \angle BAD = 90°$ $\angle B + \angle C = 90°$ $\angle CAD + \angle C = 90°$	Subtraction Postulate
$\angle B + \angle BAD = \angle B + \angle C$ $\angle B + \angle C = \angle CAD + \angle C$	Transitive Postulate
$\angle BAD = \angle C$ $\angle B = \angle CAD$	Subtraction Postulate
$\Delta CAD \sim \Delta ABD$	AA

b. If $AC = 15$ and $AD = \frac{120}{17}$, find the area of ΔABD. Hint: If you find CD then you'll have the ratios $\frac{\text{hypotenuse}}{\text{short leg}}, \frac{\text{hypotenuse}}{\text{long leg}}, \frac{\text{long leg}}{\text{short leg}}$ for all three triangles

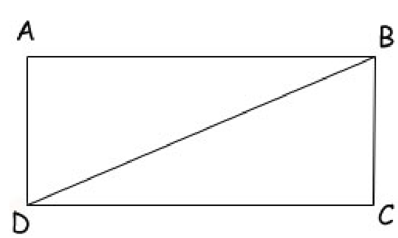
SOLN: The area of $\Delta ABD = \frac{1}{2} BD \cdot AD$, so we'd like to find *BD*, but we'll follow the hint and first find *CD*. Using the Pythagorean theorem, we have

$$CD^2 = 15^2 - \left(\frac{120}{17}\right)^2 = \left(15 - \frac{120}{17}\right)\left(15 + \frac{120}{17}\right) = \frac{255 - 120}{17} \cdot \frac{155 + 120}{17} = \frac{135 \cdot 275}{17^2}$$
$$= \frac{4125}{289}$$

Now, $CD = \frac{5\sqrt{33}}{17}$. Let $x = BD$. Now equating $\frac{\text{long leg}}{\text{short leg}}$ in $\Delta ABD \sim \Delta CAD$ $\frac{120}{17x} = \frac{5(17)\sqrt{33}}{17(120)}$, or simplifying, $\frac{120}{17x} = \frac{\sqrt{33}}{24}$ and so $x = \frac{(120)(24)}{17\sqrt{33}} = \frac{960\sqrt{33}}{187}$.

Finally, the area $\Delta ABD = \frac{1}{2} BD \cdot AD = \frac{1}{2} \cdot \frac{960\sqrt{33}}{187} \cdot \frac{120}{17} = \frac{57600\sqrt{33}}{1309}$. Wow, that was numerically challenging!

7. Consider quadrilateral *ABCD* shown at right and suppose we know that $\angle ADB \cong \angle CBD$. What type of quadrilateral can we deduce that *ABCD* is?

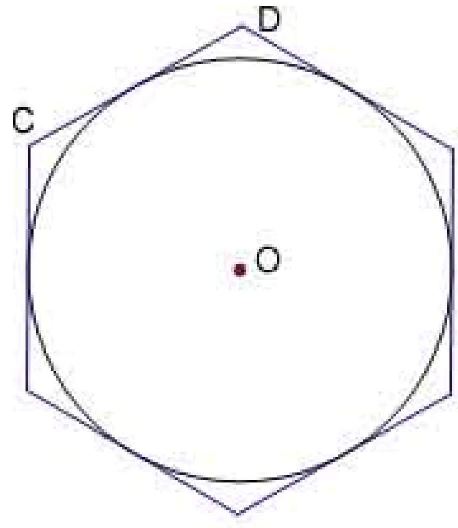


SOLN: When transversal $\overline{BD}$ cut lines $\overline{AD}$ and $\overline{BC}$, it makes alternate interior angles equal, so we can deduce that $\overline{AD} || \overline{BC}$ so that *ABCD* is either a trapezoid (if $\overline{AB} \nparallel \overline{DC}$) or a parallelogram (if $\overline{AB} || \overline{DC}$).

8. Consider the circle inscribed in a regular hexagon, as shown in the diagram to the right.

    a. What is the measure of central angle $\angle COD$?
    SOLN: $\frac{360°}{6} = 60°$

    b. If $CD = 8$, what is the area of the circle?
    SOLN: ΔCOD is equilateral, so its altitude, $4\sqrt{3}$, is the radius of the circle, so the area of the circle is $\pi(4\sqrt{3})^2 = 48\pi$.



I was interested in how students' homework scores on ILRN correlated with their test scores, so I produced this scatterplot (the outliers are removed) showing a significant positive correlation.