Math 5 – Trigonometry – Chapter 11 Test – Fall '12 Name________________________

Show your work for credit. Write all responses on separate paper. Do not abuse a calculator.

1. Find an equation for the parabola with vertex at (0,0) and
   a. focus at (0,9).
   b. directrix along $x = 2$.
   c. passing through (2, 3).
   d. focal diameter from (3, –8) to (3, 8).

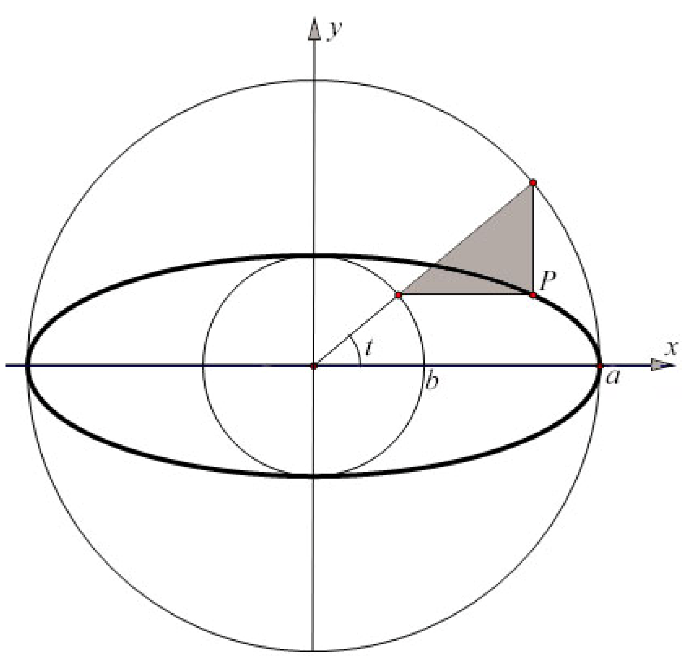
2. Find an equation for the ellipse with foci $(\pm 8, 0)$ and
   a. (minor axis) vertices $(0, \pm 12)$
   b. passing through (8,4).
   c. eccentricity = 0.5

3. Find the vertices, foci, and asymptotes of the hyperbola $108y^2 - 75x^2 = 300$ and sketch a graph illustrating these features.

4. Find parametric equations to describe the conic section:
   a. $4x^2 + y^2 = 1$
   b. $16x^2 - 9y^2 = 100$

5. Write the equation for the conic section described by $\begin{matrix} y = 2\sec(3t) \\ x = 5\tan(3t) \end{matrix}$ in rectangular form.

6. Consider the diagram below with concentric circles with radii a and b where $a > b$ centered at the origin and the angle t swept out counterclockwise from the positive x-axis.
   a. Using $\cos^2\theta + \sin^2\theta = 1$, prove that $x = a\cos t, y = b\sin t$ parameterizes $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
   b. Note that for any t, the shaded right triangle below has hypotenuse $a - b$. Show that the coordinates of point P are $(a\cos t, b\sin t)$.



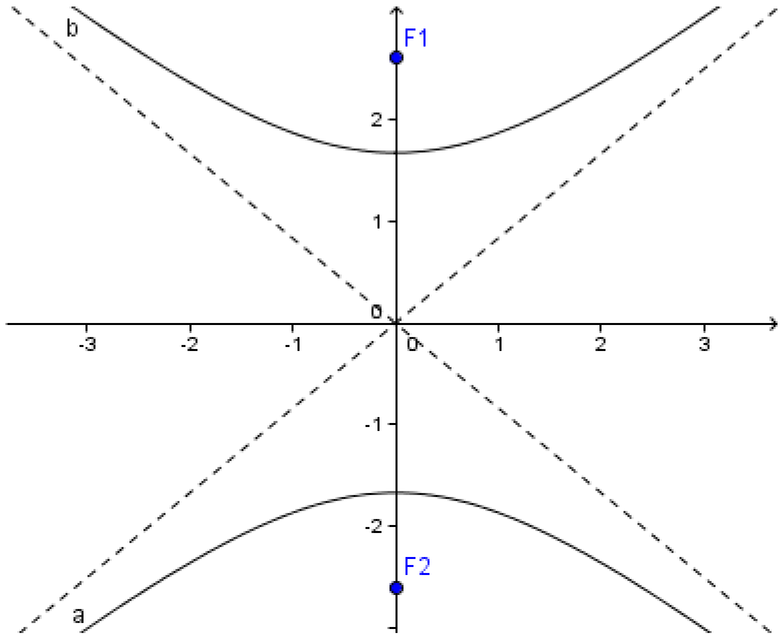
Math 5 – Trigonometry – Chapter 11 Test Solutions.

1. Find an equation for the parabola with vertex at (0,0) and
   c. focus at (0,9).
      SOLN: $4py = x^2$ leads to $36y = x^2$.
   d. directrix along $x = 2$.
      SOLN: $4px = y^2$ leads to $4(-2)x = y^2$, or, equivalently, $-8x = y^2$
   e. passing through (2, 3).
      SOLN: There are two such parabolas: one horizontal and one vertical. Specifically,
      $4p(3) = (2)^2$ leads to $4y = 3x^2$ and $4p(2) = (3)^2$ leads to $9x = 2y^2$.
   f. latus rectum from (3, –8) to (3, 8).
      SOLN: This means that the length of the latus rectum, $4p = 16$ so $p = 4$.
      Since the parabola opens to the right, $16x = y^2$.

2. Find an equation for the ellipse with foci $(\pm 8, 0)$ and
   a. (minor axis) vertices $(0, \pm 12)$
      SOLN: $c = 8$ and $b = 12$ means that $a^2 = 8^2 + 12^2 = 208$ so the standard form of the equation is $x^2/208 + y^2/144 = 1$
   b. passing through (8,4).
      SOLN: $a^2 = 8^2 + b^2$ means we can write
      $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{64}{64+b^2} + \frac{16}{b^2} = 1 \Leftrightarrow 64b^2 + 16\left(64+b^2\right) = b^2\left(64+b^2\right) \Leftrightarrow b^4 - 16b^2 = 1024$$
      $$\Leftrightarrow b^4 - 16b^2 + 64 = 1024 + 64 \Leftrightarrow \left(b^2 - 8\right)^2 = 1088 \Leftrightarrow b^2 = 8 \pm \sqrt{1088} \text{ so}$$
      $b^2 = 8 + 8\sqrt{17} \Rightarrow a^2 = 72 + 8\sqrt{17}$ and the equation is $\boxed{\dfrac{x^2}{72+8\sqrt{17}} + \dfrac{y^2}{8+8\sqrt{17}} = 1}$
   c. eccentricity = 0.5
      SOLN: Eccentricity = $c/a = 8/a = 1/2$ means $a = 16$ and $a^2 = 64 + b^2$ means $b^2 = 192$ so the equation is $\boxed{\dfrac{x^2}{256} + \dfrac{y^2}{192} = 1}$

3. Find the vertices, foci, and asymptotes of the hyperbola $108y^2 - 75x^2 = 300$ and sketch a graph illustrating these features.

   SOLN: $108y^2 - 75x^2 = 300 \Leftrightarrow \dfrac{9y^2}{25} - \dfrac{x^2}{4} = 1$ so $a = \dfrac{5}{3}$, $b = 2$ and $c^2 = \left(\dfrac{5}{3}\right)^2 + 2^2 = \dfrac{61}{9}$. The vertices are at $\left(0, \pm\dfrac{5}{3}\right)$ the foci are at $\left(0, \pm\dfrac{\sqrt{61}}{3}\right)$. The asymptotes are $y = \pm\dfrac{5}{6}x$ and the graph is shown:



4. Find parametric equations to describe the conic section:
   a. $4x^2 + y^2 = 1 \Leftrightarrow \boxed{x = \sin(17t)/2;\ \ y = \cos(17t)}$
   b. $16x^2 - 9y^2 = 100 \Leftrightarrow \frac{4x^2}{25} - \frac{9y^2}{100} = 1 \Leftrightarrow x = \frac{5}{2}\sec t\,, y = \frac{10}{3}\tan t$
5. in rectangular form, the system $\begin{matrix} y = 2\sec(3t) \\ x = 5\tan(3t) \end{matrix}$ is $\frac{y^2}{4} - \frac{x^2}{25} = 1$
6. Consider the diagram below with concentric circles with radii a and b where $a > b$ centered at the origin and the angle t swept out counterclockwise from the positive x-axis.
   a. Using $\cos^2\theta + \sin^2\theta = 1$, prove that $x = a\cos t\,, y = b\sin t$ parameterizes $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

      Proof: Substituting, se have $\frac{(a\cos t)^2}{a^2} + \frac{(b\sin t)^2}{b^2} = \cos^2 t + \sin^2 t = 1$, by Pythagoras' identity.
   b. Note that for any t, the shaded right triangle below has hypotenuse $a - b$. Show that the coordinates of point P are $(a\cos t\,, b\sin t)$.
      SOLN: The x-coordinate of P is the same as the x-coordinate on the circle of radius a, which is $a\cos t$ while the y-coordinate of P is the same as the y-coordinate on the circle of radius b, which is $b\sin t$.