Math 5 – Trigonometry – Take Home Test Problems for Chapter 11.

Show your work for credit. Give as much detail in your answer as you can.

1. Consider the conic described by $r = \frac{2}{3+3cos\theta}$
   a. Write the conic in standard polar form and determine whether it is a parabola, ellipse or hyperbola.
   b. Find the directrix of the conic.
   c. Sketch a graph of the conic showing key features such as vertices, foci, asymptotes, etc.
   d. Find the necessary parameters and write the conic in rectangular form.
   e. Give parametric equations for the conic.

2. Consider the conic described by $r = \frac{12}{6-3cos\theta}$
   a. Write the conic in standard polar form and determine whether it is a parabola, ellipse or hyperbola.
   b. Find the directrix of the conic.
   c. Sketch a graph of the conic showing key features such as vertices, foci, asymptotes, etc.
   d. Find the necessary parameters and write the conic in rectangular form.
   e. Give parametric equations for the conic.

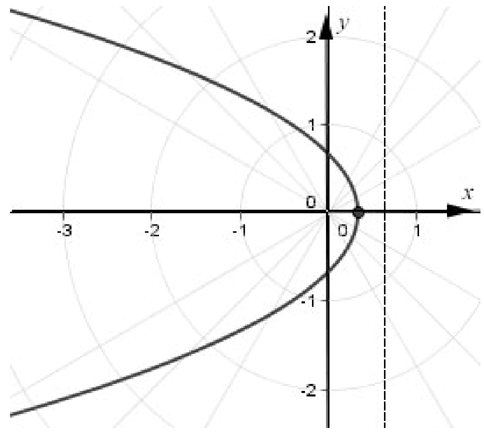
3. Consider the conic described by $r = \frac{24}{4-8\sin\theta}$
   a. Write the conic in standard polar form and determine whether it is a parabola, ellipse or hyperbola.
   b. Find the directrix of the conic.
   c. Sketch a graph of the conic showing key features such as vertices, foci, asymptotes, etc.
   d. Find the necessary parameters and write the conic in rectangular form.
   e. Give parametric equations for the conic.

Math 5 – Trigonometry – Take Home Test Problems Solutions for Chapter 11.

1. Consider the conic described by $r = \frac{2}{3+3cos\theta}$
   a. Write the conic in standard polar form and determine whether it is a parabola, ellipse or hyperbola.
      SOLN: $r = \frac{2}{3+3cos\theta} = \frac{2/3}{1+\cos\theta}$ means this is a parabola.
   b. Find the directrix of the conic.
      SOLN: The form of the equation is $r = \frac{ed}{1+ecos\theta} = \frac{2/3}{1+\cos\theta}$ so $e = 1$ and $d = \frac{2}{3}$ so the directrix is $x = \frac{2}{3}$
   c. Sketch a graph of the conic showing key features such as vertices, foci, asymptotes, etc.
      SOLN: Here *d* = the distance from the focus to the directrix. So the vertex is at $\left(\frac{1}{3}, 0\right)$, the focus is at (0,0), the parabola opens to the left and the focal diameter has length 4/3, from $(0, -\frac{2}{3})$ to $\left(0, \frac{2}{3}\right)$.
      Tabulating values, we have

θ	0	$\pm\pi/3$	$\pm\pi/2$	$\pm 2\pi/3$
r	1/3	4/9	2/3	4/3

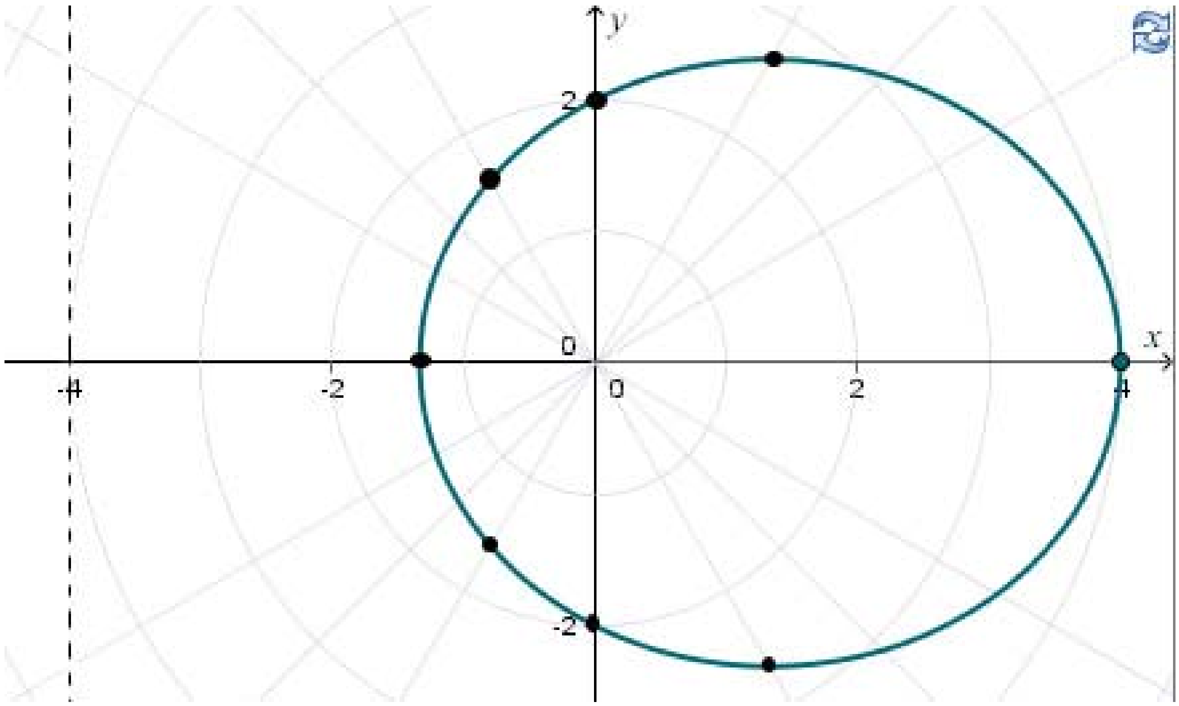
The graph below was developed using the Geogebra applet at
http://webspace.ship.edu/msrenault/ggb/polar_grapher.html



   d. Find the necessary parameters and write the conic in rectangular form.
      SOLN: Using the vertex form: $x = h + a(y-k)^2$ and the vertex $(h,k) = \left(\frac{1}{3}, 0\right)$ we get $x = \frac{1}{3} + ay^2$ and since the parabola passes through $\left(0, \frac{2}{3}\right)$, $0 = \frac{1}{3} + \frac{4}{9}a \Leftrightarrow a = -\frac{3}{4}$ and so $x = \frac{1}{3} - \frac{3}{4}y^2$.
   e. Give parametric equations for the conic.
      SOLN: Just let $y = t, x = \frac{1}{3} - \frac{3}{4}t^2$

2. Consider the conic described by $r = \frac{12}{6-3cos\theta}$
   a. Write the conic in standard polar form and determine whether it is a parabola, ellipse or hyperbola.
      SOLN: In standard form, $r = \frac{2}{1-\frac{1}{2}cos\theta}$, so this is an ellipse with eccentricity, $e = \frac{1}{2}$.
   b. Find the directrix of the conic.
      SOLN: Since $ed = \frac{1}{2} \cdot d = 2$, $d = 4$ and the directrix is $x = -4$.
   c. Sketch a graph of the conic showing key features such as vertices, foci, asymptotes, etc.
      Make a table of values

θ	0	$\pm\pi/3$	$\pm\pi/2$	$\pm 2\pi/3$	$\pm\pi$
r	4	8/3	2	8/5	4/3



d. Find the necessary parameters and write the conic in rectangular form.

SOLN: The major axis extends from $\left(-\frac{4}{3}, 0\right)$ to $(4,0)$ and so the center is at $x = \frac{-\frac{4}{3}+4}{2} = \frac{4}{3}$ and the distance from the center to a focus is $c = \frac{4}{3}$, while the distance from the center to a vertex is $a = \frac{8}{3}$. Thus $b^2 = a^2 - c^2 = \frac{64}{9} - \frac{16}{9} = \frac{16}{3}$. The rectangular form is then

$$\frac{\left(x-\frac{4}{3}\right)^2}{\frac{64}{9}} + \frac{y^2}{\frac{16}{3}} = 1 \Leftrightarrow \frac{9\left(x-\frac{4}{3}\right)^2}{64} + \frac{3y^2}{16} = 1$$

e. Give parametric equations for the conic.

SOLN: $x = \frac{4}{3} + \frac{8}{3}\cos t\,, y = \frac{4\sqrt{3}}{3}\sin t$

3. Consider the conic described by $r = \frac{24}{4-8\sin\theta}$

a. Write the conic in standard polar form and determine whether it is a parabola, ellipse or hyperbola.

SOLN: $r = \frac{6}{1-2\sin\theta}$ has $e = 2$, so it's a hyperbola.

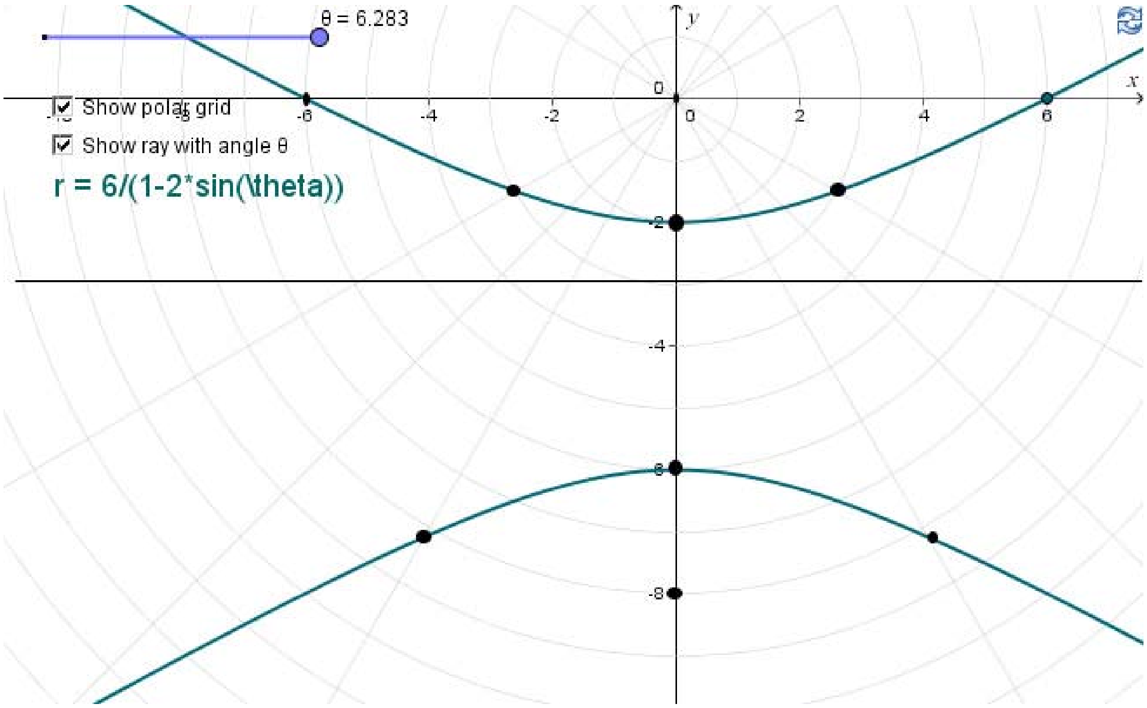
b. Find the directrix of the conic.

SOLN: Here $ed = 2d = 6$ so $d = 3$ and the directrix is $y = -3$.

Sketch a graph of the conic showing key features such as vertices, foci, asymptotes, etc.

SOLN: Make a table of values

θ	0	$\pi/3$	$\pi/2$	$2\pi/3$	π	$7\pi/6$	$3\pi/2$	$11\pi/6$
r	6	$-3(1+\sqrt{3}) \approx -8.2$	-6	$-3(1+\sqrt{3}) \approx -8.2$	6	3	2	3



c. Find the necessary parameters and write the conic in rectangular form.
SOLN: The center is at $(h,k) = (0,-4)$ with vertices at $(0,-2)$ and $(0,-6)$ so $a = 2$ and $c = 4$ giving us $b^2 = 16 - 4 = 12$. Thus the equation is $\frac{(y+4)^2}{4} - \frac{x^2}{12} = 1$

d. Give parametric equations for the conic.
SOLN: $x = 2\sqrt{3}\tan t$, $y = -4 + 2\sec t$