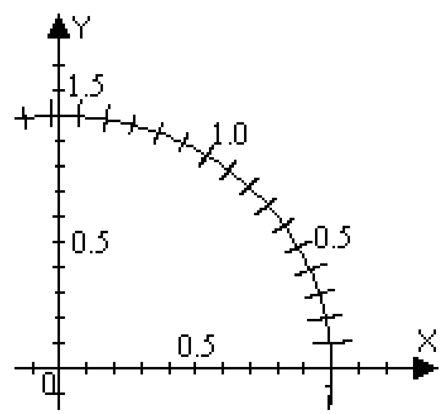
Math 5 – Trigonometry – Chapter 4 Test – Fall ’08 Name__________________________
Show work for credit. Write all responses on separate paper. Do not use a calculator.

1. Consider an arclength of $t = \frac{23\pi}{4}$ travelled counter-clockwise around the circumference of the unit circle, starting at (1,0).
   a. What quadrant is the terminal point in?
   b. What is the value of the reference number $\bar{t}$?
   c. Evaluate $\sin\left(\frac{23\pi}{4}\right)$, $\cos\left(\frac{23\pi}{4}\right)$ and $\tan\left(\frac{23\pi}{4}\right)$

2. Recall that a function is even if $f(-x) = f(x)$ and a function is odd if $f(-x) = -f(x)$.
   Of the 6 trigonometric functions: sin(x), cos(x), tan(x), csc(x), sec(x) and cot(x),
   a. Which functions are even?
   b. Which functions are odd?

3. If $\cos(t) = \frac{12}{37}$ and t leads to a terminal point in the fourth quadrant,
   a. Find sin(t)
   b. Find tan(t)

4. Consider the point $\left(\frac{5}{13}, \frac{12}{13}\right)$
   a. Verify that the point lies on the unit circle.
   b. Use the diagram at right to approximate to the nearest tenth the smallest positive value of t so that $\cos(t) = \frac{5}{13} \approx 0.38$
   c. Approximate to the nearest tenth the t-interval for which $0.38 \approx \frac{5}{13} < \cos(t) < \frac{12}{13} \approx 0.92$

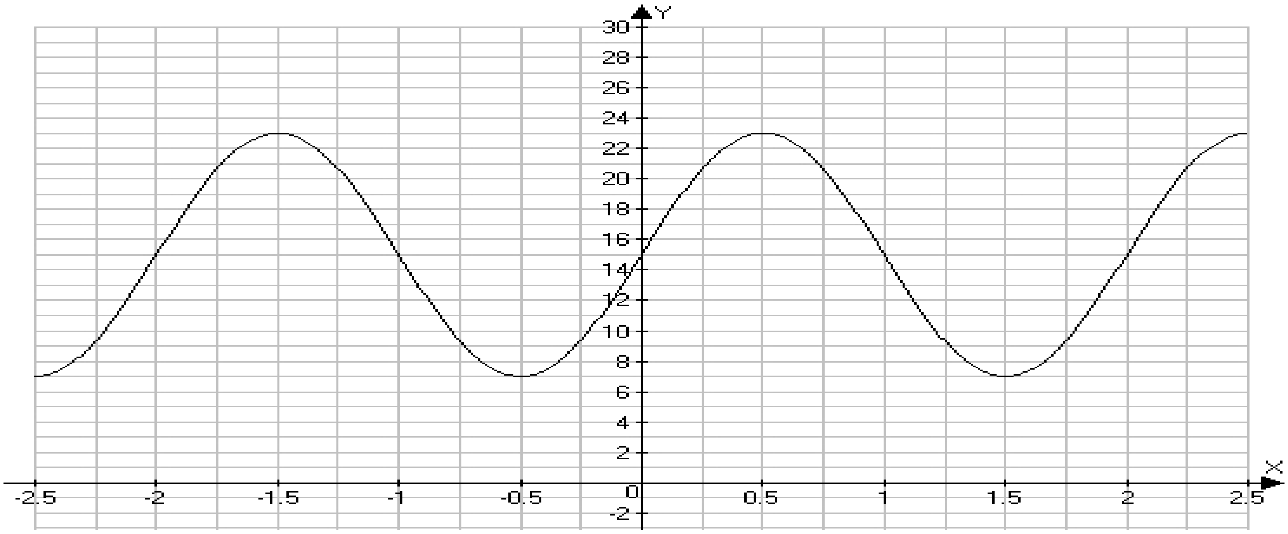


5. Find the values of the other five trigonometric functions of t if $\tan(t) = -3$ and the terminal point of t is in quadrant II. Simplify all radical expressions and rationalize denominators.
   Hint: recall the Pythagorean identy: $\cos^2 t + \sin^2 t = 1 \Leftrightarrow 1 + \tan^2 t = \sec^2 t$

6. Find the period, amplitude and phase angle of $y(t) = 6\cos(3\pi t)$ and sketch a graph showing at least 5 points on at least one complete oscillation of the sinuosoid.

7. Find the period, amplitude and phase angle of $y(t) = 6\sin\left[8(t-1)\right]$ and sketch a graph showing at least 5 points on the sinuosoid.

8. Find an equation for the sinusoid whose graph is shown:



9. Consider the function $f(x) = \tan\left(\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right)$.

   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates where $y = 0$ and where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

10. Suppose $\sin t = 16/65$ and t is in the first quadrant. Plot the following points on the unit circle and find the values of the coordinates as rational numbers:

   a. $(x, y) = (\cos(t + \pi), \sin(t + \pi))$
   b. $(x, y) = \left(\cos\left(t + \frac{\pi}{2}\right), \sin\left(t + \frac{\pi}{2}\right)\right)$
   c. $(x, y) = \left(\cos\left(\frac{\pi}{2} - t\right), \sin\left(\frac{\pi}{2} - t\right)\right)$

11. Complete the table of values for $f(t) = \cos(\pi t) + 2\sin(\pi t)$, plot the points and sketch a graph.

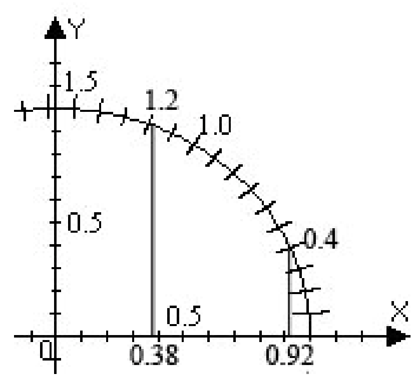
t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$\cos(\pi t)$									
$2\sin(\pi t)$									
$f(t)$									

Math 5 – Trigonometry – Chapter 4 Test Solutions – Fall '08

1. Consider an arclength of $t = \frac{23\pi}{4}$ travelled counter-clockwise around the circumference of the unit circle, starting at (1,0).
   a. What quadrant is the terminal point in?
   SOLN: $t = \frac{23\pi}{4} = 4\pi + \frac{7\pi}{4}$ is in QIV.
   b. What is the value of the reference number $\bar{t}$?
   SOLN: $\bar{t} = \frac{\pi}{4}$
   c. Evaluate $\sin\left(\frac{23\pi}{4}\right)$, $\cos\left(\frac{23\pi}{4}\right)$ and $\tan\left(\frac{23\pi}{4}\right)$
   SOLN: $\sin\left(\frac{23\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, $\cos\left(\frac{23\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, and $\tan\left(\frac{23\pi}{4}\right) = -1$
2. Recall that a function is even if $f(-x) = f(x)$ and a function is odd if $f(-x) = -f(x)$.
   Of the 6 trigonometric functions: sin(x), cos(x), tan(x), csc(x), sec(x) and cot(x),

   a. Which functions are even?
   SOLN: cos(x) and sec(x) are even functions.

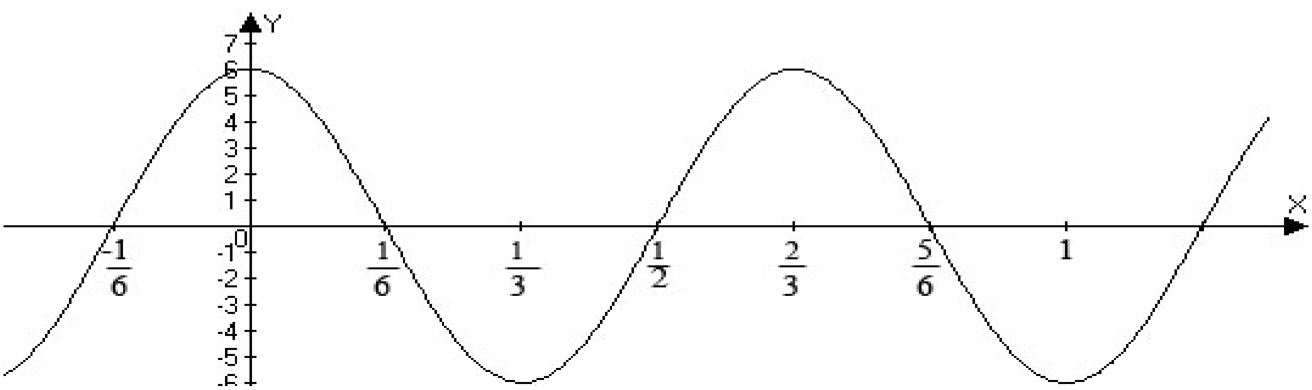
   b. Which functions are odd?
   SOLN: sin(x), tan(x), csc(x), and cot(x) are odd.
3. If $\cos(t) = \frac{12}{37}$ and t leads to a terminal point in the fourth quadrant,
   a. Find sin(t)
   SOLN: $\sin(t) = -\sqrt{1-\cos^2 t} = -\sqrt{1-\left(\frac{12}{37}\right)^2} = -\sqrt{\frac{1369-144}{1369}} = \sqrt{\frac{1225}{1369}} = \frac{-35}{37}$
   b. Find tan(t) SOLN: $\tan(t) = \frac{35/37}{-12/37} = \frac{-35}{12}$
4. Consider the point $\left(\frac{5}{13}, \frac{12}{13}\right)$
   a. Verify that the point lies on the unit circle.
   SOLN: $\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{25}{169} + \frac{144}{169} = \frac{169}{169} = 1$
   b. Use the diagram at right to approximate to the nearest tenth the smallest positive value of t so that $\cos(t) = \frac{5}{13} \approx 0.38$
   SOLN: If $\cos(t) \approx 0.38$ in QI then $t \approx 1.2$ (diagram)



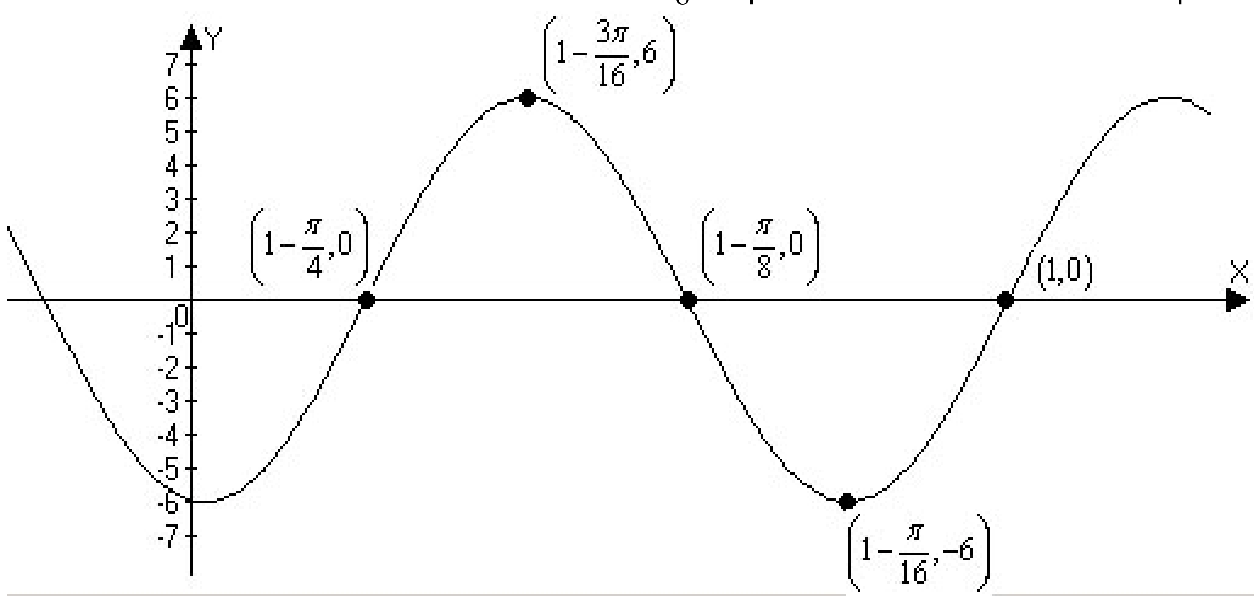
   c. Approximate to the nearest tenth the t-interval for which $0.38 \approx \frac{5}{13} < \cos(t) < \frac{12}{13} \approx 0.92$
   SOLN: As shown in the diagram, if cos(t) is approximately between 0.38 and 0.92, then t is approx. between 0.4 and 1.2, though, since cos(t) is decreasing in QI, the larger t goes with the smaller cos(t) and vice versa.

5. Find the values of the other five trigonometric functions of t if $\tan(t) = -3$ and the terminal point of t is in quadrant II. Simplify all radical expressions and rationalize denominators.
Hint: recall the Pythagorean identy: $\cos^2 t + \sin^2 t = 1 \Leftrightarrow 1 + \tan^2 t = \sec^2 t$
SOLN: cot(t) = –1/3, $1 + \tan^2 t = 1 + (-3)^2 = \sec^2 t \Leftrightarrow \sec^2 t = 10 \Rightarrow \sec t = -\sqrt{10}$, since sec($t$) is negative in QII.
Thus, $\cos(t) = \frac{1}{-\sqrt{10}} = \frac{-\sqrt{10}}{10}$, $\sin(t) = \sqrt{1-\cos^2(t)} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$ and $\csc(t) = \frac{1}{\sin(t)} = \frac{\sqrt{10}}{3}$.
That's all six.

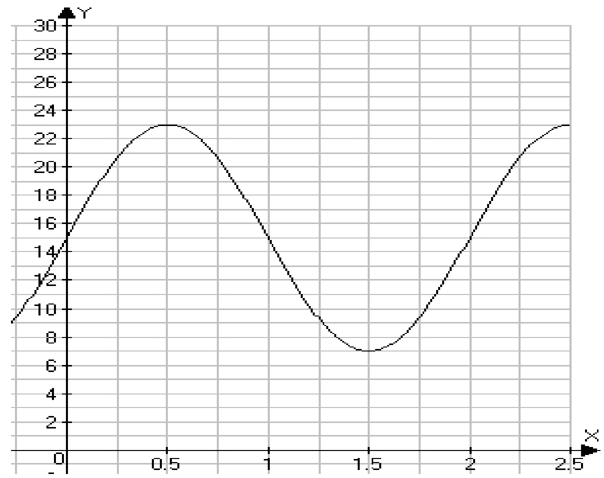
6. Find the period, amplitude and phase angle of $y(t) = 6\cos(3\pi t)$ and sketch a graph showing at least 5 points on at least one complete oscillation of the sinusoid.
SOLN: The period is $\frac{2\pi}{3\pi} = \frac{2}{3}$, the amplitude is 6 and the phase angle is zero or $-\frac{1}{6}$ depending on whether you're thinking of the cosine or the sine as your basic waveform.



7. Find the period, amplitude and phase angle of $y(t) = 6\sin[8(t-1)]$ and sketch a graph showing at least 5 points on the sinusoid. SOLN: period = $\frac{2\pi}{8} = \frac{\pi}{4}$, amp = 6 and phase shift = $1 - \frac{\pi}{4}$.



8. Find an equation for the sinusoid whose graph is shown:



ANS: The lowest point is at y=7 and the highest point is at 23 so the line of equilibrium is at the average of these: y = (7+23)/2 = 15. and the amplitude is (23 – 7)/2 = 8.

The two peaks shown in the graph here are where x = 0.5 and x = 2.5, so the period is 2.5 – 0.5 = 2.
Thus an equation for the sinusoid is
$y = 15 + 8\sin(\pi x)$.

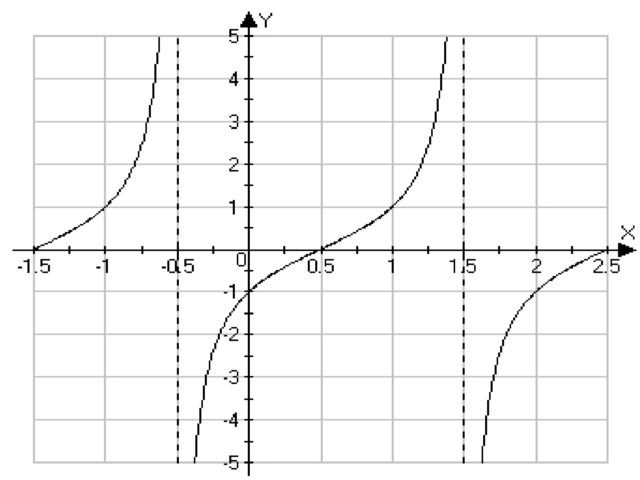
9. Consider the function $f(x) = \tan\left(\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right)$.

a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.

ANS: We want the input to the tangent to be $\pm\frac{\pi}{2}$, that is

$$\frac{\pi}{2}\left(x-\frac{1}{2}\right) = \pm\frac{\pi}{2} \Leftrightarrow x - \frac{1}{2} = \pm 1 \Leftrightarrow \boxed{x = \frac{1}{2} \pm 1 = -\frac{1}{2} \text{ or } \frac{3}{2}}$$

b. Find x-coords where $y = 0$ and $y = \pm 1$.

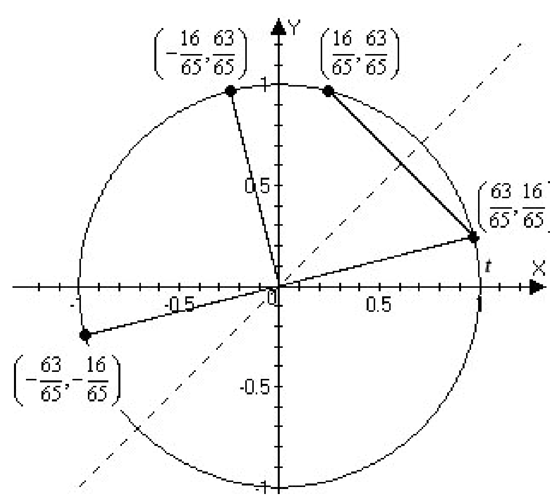


ANS: We want to find where the input to the tangent function is equal to $\pm\frac{\pi}{4}$, that is $\frac{\pi}{2}\left(x-\frac{1}{2}\right) = \pm\frac{\pi}{4} \Leftrightarrow x - \frac{1}{2} = \pm\frac{1}{2}$.

$$\Leftrightarrow \boxed{x = 0 \text{ or } x = 1}$$

c. Graph of the function showing how it passes through the points where y = -1, y = 0, y = 1 and how it approaches the vertical asymptotes.

10. Suppose *sin t* = 16/65 and t is in the first quadrant. Find the following:



a. At $t+\pi$, $(x, y) = \left(-\frac{63}{65}, -\frac{16}{65}\right)$

b. At $t + \frac{\pi}{2}$, $(x, y) = \left(-\frac{16}{65}, \frac{63}{65}\right)$

c. At $\frac{\pi}{2} - t$, $(x, y) = \left(\frac{16}{65}, \frac{63}{65}\right)$

11. Complete the table of values for $f(t)=\cos(\pi t)+2\sin(\pi t)$, plot the points and sketch a graph.

t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$\cos(\pi t)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$2\sin(\pi t)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0
$f(t)$	1	$1+\frac{\sqrt{3}}{2}$	$\frac{3\sqrt{2}}{2}$	$\frac{1}{2}+\sqrt{3}$	2	$-\frac{1}{2}+\sqrt{3}$	$\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{3}}{2}$	-1