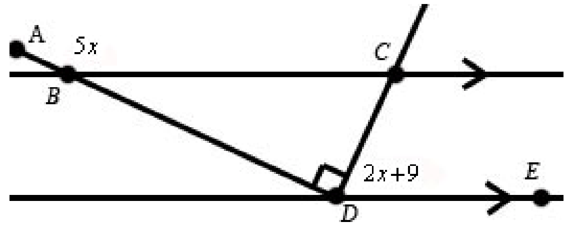
Math 5 – Trigonometry – Chapter 1 Test – Spring ’09 Name________________________________
Show your work for credit. Write all responses on separate paper. Do not use a calculator.

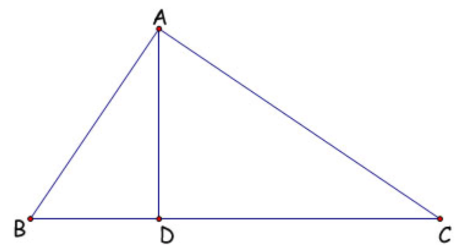
1. In the figure at right, $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$.
   What is the degree measure of x so that $\angle ABC = 5x$ and $\angle CDE = 2x+9$?



2. Find the altitude of an equilateral triangle with sides of length 10.

3. Draw an isosceles right triangle with hypotenuse of length 2 inches. Find the perimeter and area of this triangle.

4. Consider the diagram at right and assume that $\overline{AB} \perp \overline{AC}$ and that $\overline{AD} \perp \overline{BC}$.

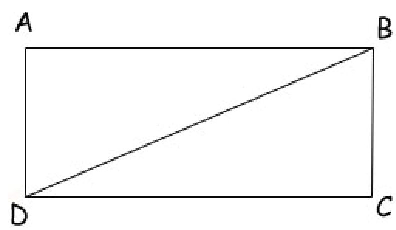


   a. Prove that $\Delta CAD \sim \Delta ABD$

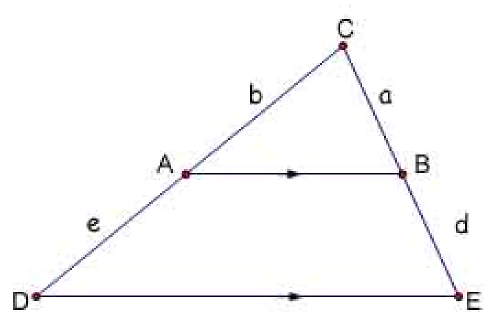
   b. If $AC = 15$ and $AD = \frac{120}{17}$, find the area of ΔABD.
   Hint: If you find CD then you’ll have the ratios $\frac{\text{hypotenuse}}{\text{short leg}}, \frac{\text{hypotenuse}}{\text{long leg}}, \frac{\text{long leg}}{\text{short leg}}$ for all three triangles

5. Deduce the truth of the theorem stating that the sum of the interior angles of any triangle is 180 degrees. You might start by drawing a line through one of the vertices that is parallel to side opposite that vertex.

6. Consider quadrilateral $ABCD$ shown at right and suppose we know that $\angle ADB \cong \angle CBD$.
   What type of quadrilateral can we deduce that $ABCD$ is?



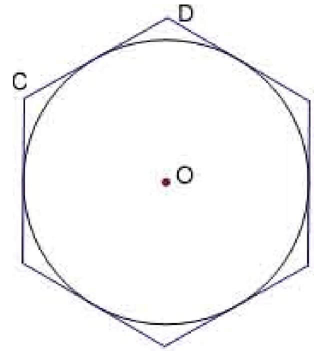
7. Refer to the diagram to the right, where $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$.
   a. Prove that $\Delta CAB \sim \Delta CDE$
   b. Prove that $\frac{a}{b} = \frac{d}{e}$.
   c. Use this result to prove that the base angles of an isosceles trapezoid are congruent.



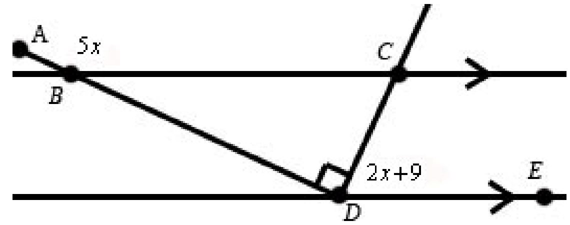
8. Consider the circle inscribed in a regular hexagon, as shown in the diagram to the right.

   a. What is the measure of central angle $\angle COD$?

b. If $CD = 8$, what is the area of the circle?



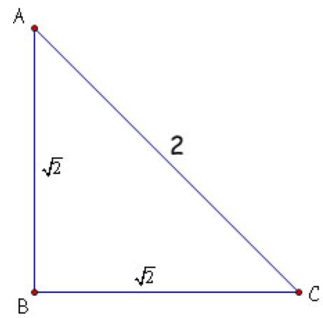
Math 5 – Trigonometry – Chapter 1 Test Solutions – Spring '09



1. In the figure at right, $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$.
   What is the degree measure of x so that $\angle ABC = 5x$ and $\angle CDE = 2x+9$?

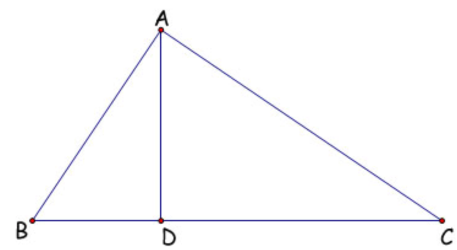
   Solution: Since $\angle ABC$ and $\angle ADE$ are corresponding angles for the transversal AD cutting parallels $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$ so that $\angle ABC = \angle ADE \Leftrightarrow 5x = 90 + 2x + 9 \Leftrightarrow 3x = 99 \Leftrightarrow x = 33^\circ$

2. Find the altitude of an equilateral triangle with sides of length 10.
   SOLN: Draw the perpendicular bisector of the base to form a 30-60-90 triangle. Then the height (altitude) of the triangle is the square root of 3 time the short leg: $h = 5\sqrt{3}$



3. Draw an isosceles right triangle with hypotenuse of length 2 inches. Find the perimeter and area of this triangle.

   SOLN: Let the length of a leg be x so that
   $x^2 + x^2 = 2^2 \Leftrightarrow x^2 = 2 \Rightarrow x = \sqrt{2}$ so
   the perimeter = $2\sqrt{2} + 2 = 2\left(1+\sqrt{2}\right)$ and
   the area = $\frac{1}{2}\left(\sqrt{2}\sqrt{2}\right) = 1$



4. Consider the diagram at right and assume that $\overline{AB} \perp \overline{AC}$ and that $\overline{AD} \perp \overline{BC}$.
   a. Prove that $\Delta CAD \sim \Delta ABD$
      SOLN: Since the sum of the interior angles of a triangle is 180 degrees and there are right angles at A and D we have that $\angle B + \angle C = 90$,
      $\angle B + \angle BAD = 90$ and $\angle CAD + \angle C = 90$ whence
      $\angle C = \angle BAD$ and $\angle B = \angle CAD$ so the triangles are equiangular which is another word for similar.

5. If $AC = 15$ and $AD = \frac{120}{17}$, find the area of ΔABD. Hint: If you find CD then you'll have the ratios $\frac{\text{hypotenuse}}{\text{short leg}}, \frac{\text{hypotenuse}}{\text{long leg}}, \frac{\text{long leg}}{\text{short leg}}$ for all three triangles

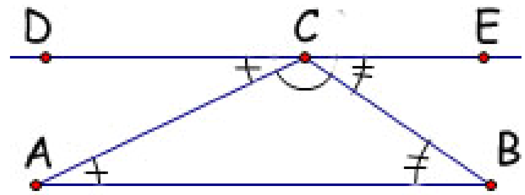
   SOLN: $\frac{\text{short}}{\text{hypotenuse}} = \frac{120}{15 \cdot 17} = \frac{120}{255} = \frac{8}{17} = \frac{BD}{AB} = \frac{AB}{BD+CD} = \frac{AB}{\frac{8AB}{17} + CD}$ whence

   $\frac{CD}{AB} = \frac{17}{8} - \frac{8}{17} = \frac{289-64}{136} - \frac{225}{136} = \frac{15^2}{8 \cdot 17}$. But that doesn't help a lot, does it?

   So find CD via the Pythagorean formula: $CD^2 = 15^2 - \left(\frac{120}{17}\right)^2 = \frac{255^2}{289} - \frac{14400}{289} = \frac{50625}{289} = \left(\frac{15^2}{17}\right)^2$

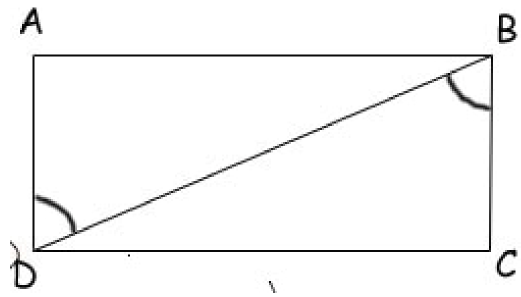
   so $CD = \frac{15^2}{17}$ and so $AB = \frac{8 \cdot 17 CD}{15^2} = \frac{8 \cdot 17}{15^2}\frac{15^2}{17} = 8$. So the area of the triangle is
   $\frac{1}{2} AB \cdot AC = \frac{1}{2}(8)(15) = 60$

6. Deduce the truth of the theorem stating that the sum of the interior angles of any triangle is 180 degrees. You might start by drawing a line through one of the vertices that is parallel to side opposite that vertex.



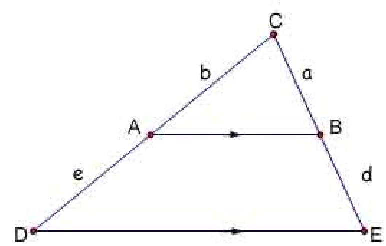
SOLN: Draw *DE* through *C* and parallel to *AB*. Then $\angle DCA = \angle A$ and $\angle ECB = \angle B$ are pairs of alternate interior angles formed when the transversals AC and BC cross the parallel lines. Since $\angle DCE$ is a straight angle and equal to the sum of its parts, $\angle DCE = \angle DCA + \angle ACB + \angle BCE = 180^\circ$ and the result follows by substitution $\angle DCA + \angle ACB + \angle BCE = \angle A + \angle ACB + \angle B = 180^\circ$

7. Consider quadrilateral *ABCD* shown at right and suppose we know that $\angle ADB \cong \angle CBD$.



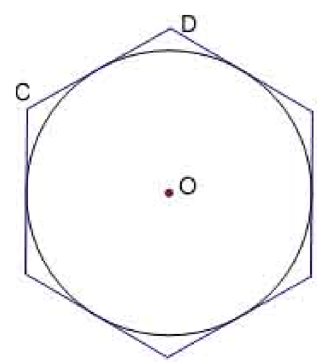
What type of quadrilateral can we deduce that *ABCD* is?
SOLN: Since transversal *BD* cuts lines *AD* and *BC* making alternate interior angles equal, we can conclude that *AD* is parallel to *BC*. However, *AB* is not necessarily parallel to *CD*, so the strongest conclusion we can draw is that *ABCD* is a trapezoid.

8. Refer to the diagram to the right, where $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$.



   a. Prove that $\Delta CAB \sim \Delta CDE$
   SOLN: Since transversals *CD* and *CE* cut parallels AB and DE, the corresponding angles $\angle CAB = \angle D$ & $\angle CBA = \angle E$ so $\Delta CAB \sim \Delta CDE$
   b. Prove that $\frac{a}{b} = \frac{d}{e}$.
   SOLN: Corresponding parts of similar triangles proportional so
   $\frac{b+e}{b} = \frac{a+d}{a} \Leftrightarrow a(b+e) = b(a+d) \Leftrightarrow ab + ae = ab + bd \Leftrightarrow \frac{ae}{be} = \frac{bd}{be} \Leftrightarrow \frac{a}{b} = \frac{d}{e}$
   c. Use this result to prove that the base angles of an isosceles trapezoid are congruent.
   SOLN: Given isosceles trapezoid ABED with AD = BE and AB parallel to DE but AD not parallel to BE, extend AD and EB until they meet at C.

9. Consider the circle inscribed in a regular hexagon, as shown in the diagram to the right.



   a. What is the measure of central angle $\angle COD$?
   SOLN: Since hexagon is regular, its diagonals divide it into 6 equilateral triangles so that each central angle, including $\angle COD = 60^\circ$
   b. If *CD* = 8, what is the area of the circle?
   SOLN: Since the triangles are equilateral, the perpendicular bisector of CD forms a 30-60-90 triangle whose altitudes have lengths $= 4\sqrt{3}$, which is then the radius of the circle, whence the circle's area is $\pi\left(4\sqrt{3}\right)^2 = 48\pi$