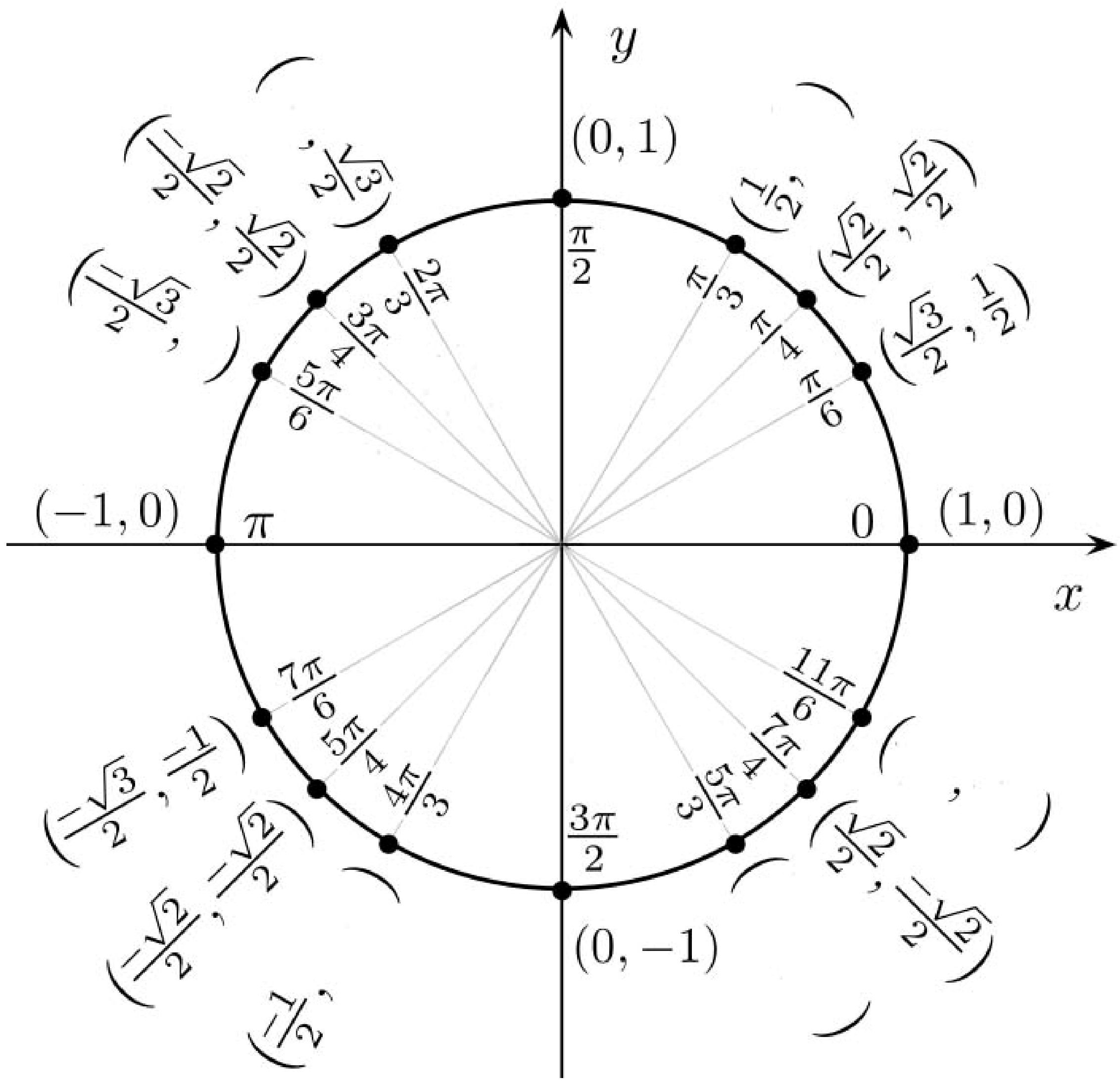
Math 5 – Trigonometry – Chapter 4 Test – fall '10 Name_________________________

Show work for credit. Unless otherwise directed, write responses on separate paper. Don't use a calculator.

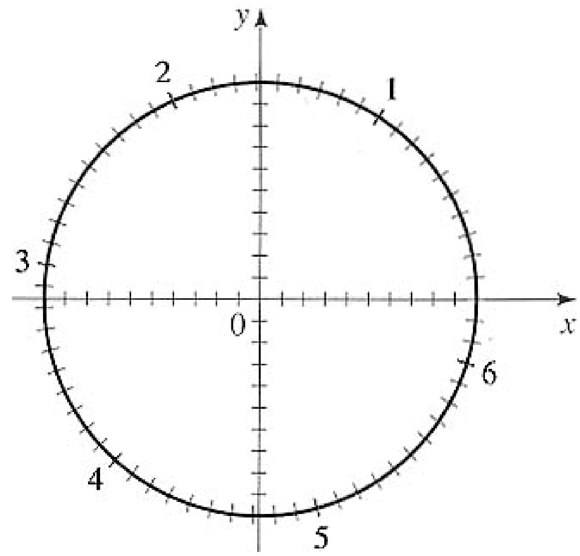
1. There are 8 blanks in the coordinates of the unit circle points illustrated below. Fill them in.



2. Find the coordinates of the terminal point determined by $t = \dfrac{41\pi}{3}$.

3. Suppose the y-coordinate of a terminal point on the unit circle is $y = \dfrac{4\sqrt{13}}{17}$ and the terminal point is in the second quadrant. Find the x-coordinate.

4. Use the diagram at right showing the number line wrapped around the circumference of the unit circle to locate the point (highlight it on the diagram here) and approximate, to the nearest tenth,
   a. $\cos(2.7)$
   b. $\sin(2.7)$
   c. $\tan(2.7)$



5. What kind of number is $\tan(4.7)$? Why?
   a. Undefined
   b. Negative of large magnitude
   c. Positive of large magnitude
   d. Negative of small magnitude
   e. Positive of small magnitude

6. Find the amplitude, period, line of equilibrium and phase shift of each function and sketch a graph clearly showing coordinates of at least 5 points of one oscillation. A table of values is useful.

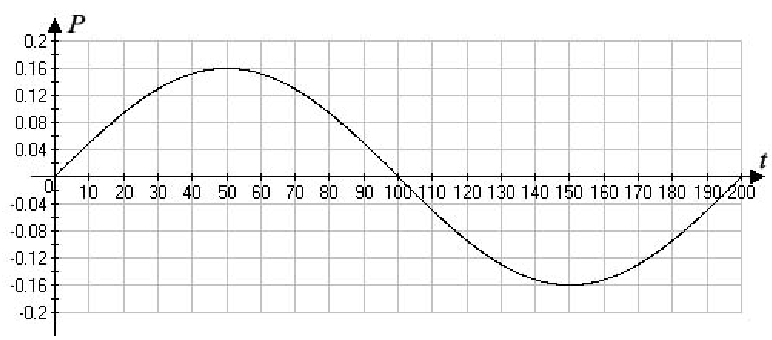
a. $y = 2 + 4\cos(3x)$	b. $y = 6 + 5\sin\left(\frac{\pi}{12}x - \frac{\pi}{6}\right)$

7. Consider the function $f(x) = \tan\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$.
   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the x-coordinates where $y = 0$ and where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

8. Suppose $\sin t = 8/17$ and t is in the first quadrant. Find the following:

a. $\sin(t - \pi)$	b. $\sin\left(t - \frac{\pi}{2}\right)$	c. $\sin\left(\frac{\pi}{2} - t\right)$

9. The graph to the right shows the air pressure, P, (in atm.) as a function of time (in milliseconds).
   a. Find the amplitude and period of the function.
   b. Give a formula for the function, as a sinusoid in t.
   c. What is the frequency of oscillation, in cycles per second?

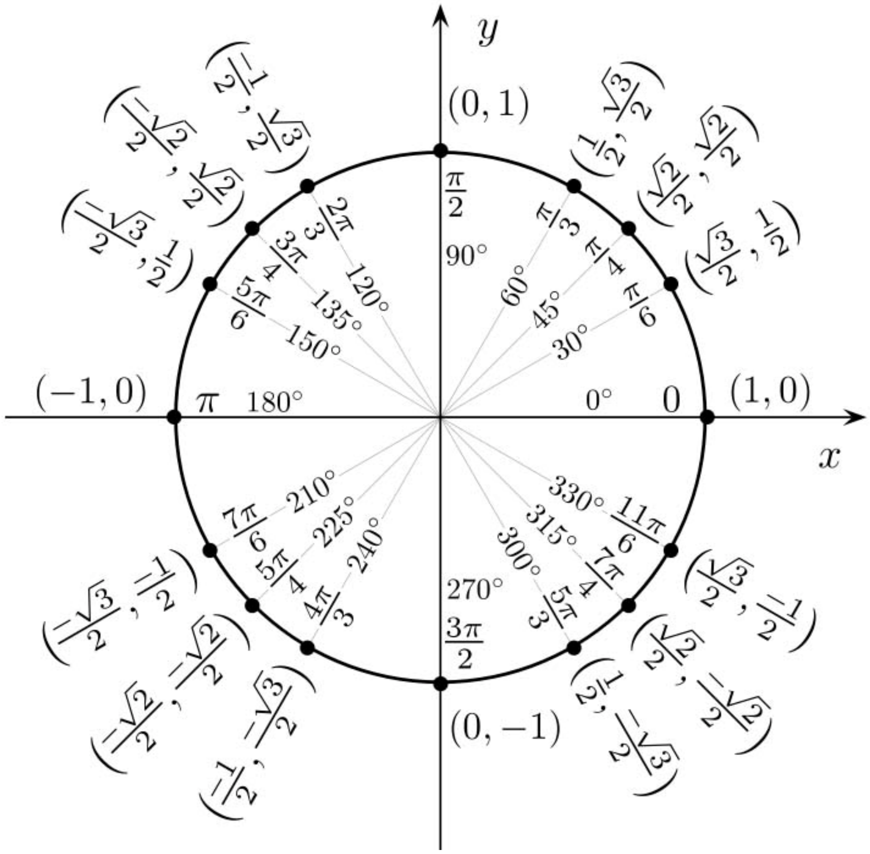


Complete the table of values for $f(t) = 2\sin(\pi t) + \sin(3\pi t)$, plot the points and sketch a graph.

t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$2\sin(\pi t)$									
$\sin(3\pi t)$									
$f(t)$									

Math 5 – Trigonometry – Chapter 4 Test Solutions – fall ’10

1. There are 8 blanks in the coordinates of the unit circle points illustrated below. Fill them in.



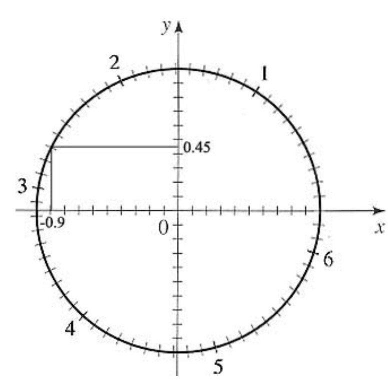
2. Find the coordinates of the terminal point determined by $t = \frac{41\pi}{3}$.

SOLN: $t = \frac{41\pi}{3} = \frac{36\pi + 5\pi}{3} = 12\pi + \frac{5\pi}{3}$ is coterminal with $\frac{5\pi}{3}$ where the coordinates are $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

3. Suppose the y-coord of a terminal point on the unit circle is $y = \frac{4\sqrt{13}}{17}$ is in the second quadrant. Find x.

SOLN: $x^2 + y^2 = x^2 + \left(\frac{4\sqrt{13}}{17}\right)^2 = x^2 + \frac{16(13)}{289} = 1 \Leftrightarrow x^2 = 1 - \frac{208}{289} = \frac{81}{289} \Leftrightarrow x = \pm\frac{9}{17}$ So $x = \frac{9}{17}$.

4. Use the diagram at right showing the number line wrapped around the circumference of the unit circle to locate the point (highlight it on the diagram here) and approximate, to the nearest tenth,
   a. $\cos(2.7) \approx -0.9$
   b. $\sin(2.7) \approx 0.45$
   c. $\tan(2.7) \approx \frac{0.45}{-0.9} = -0.5$



5. What kind of number is $\tan(4.7)$? Why?

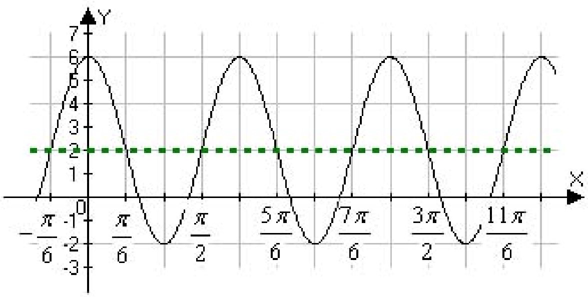
SOLN: $\tan(4.7)=\dfrac{\sin(4.7)}{\cos(4.7)}\approx\dfrac{-1}{\text{small negative}}=$ large positive

6. Find the amplitude, period, line of equilibrium and phase shift of each function and sketch a graph clearly showing coordinates of at least 5 points of one oscillation. A table of values is useful.

a. $y=2+4\cos(3x)=2+\sin\left[3\left(x+\dfrac{\pi}{6}\right)\right]$

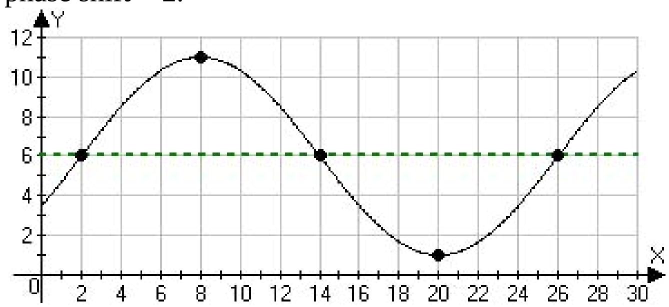
amplitude = 4, period = $\dfrac{2\pi}{3}$, equ: y = 2 and

phase shift = 0 (or $-\dfrac{\pi}{6}$ for a sine function.)



b. $y=6+5\sin\left(\dfrac{\pi}{12}x-\dfrac{\pi}{6}\right)$

amplitude = 5,
period = 24,
equ: y = 6 and
phase shift = 2.



7. Consider the function $f(x)=\tan\left(\dfrac{1}{2}\left(x-\dfrac{\pi}{4}\right)\right)$.

a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines. |
SOLN:

$$-\frac{\pi}{2}<\frac{1}{2}\left(x-\frac{\pi}{4}\right)<\frac{\pi}{2}\Leftrightarrow-\pi<x-\frac{\pi}{4}<\pi$$

$$\Leftrightarrow-\frac{3\pi}{4}<x<\frac{5\pi}{4}$$

b. Find the x-coordinates where y = 0 and where $y=\pm1$.

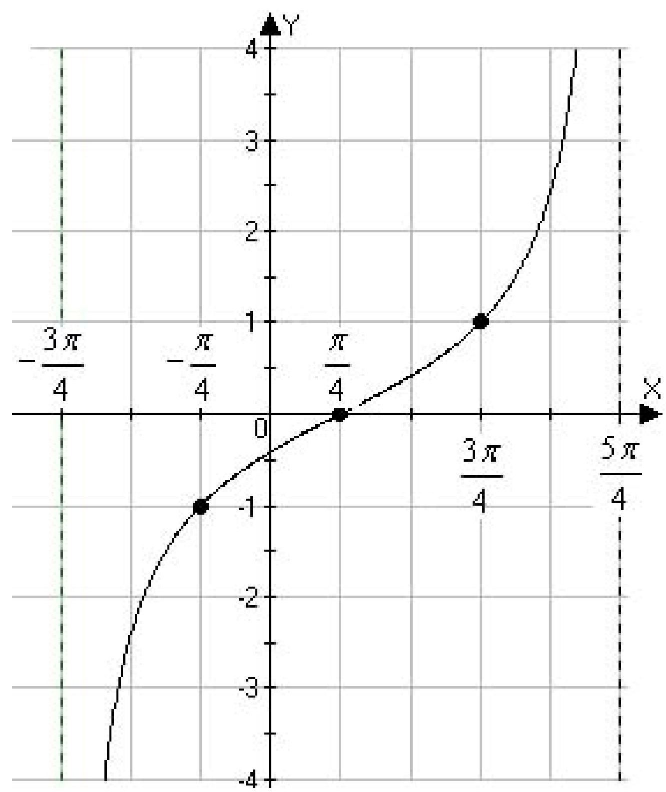
SOLN: y = 0 when $x-\dfrac{\pi}{4}=0\Leftrightarrow x=\dfrac{\pi}{4}$

$y=\pm1$ when

$$\frac{1}{2}\left(x-\frac{\pi}{4}\right)=\pm\frac{\pi}{4}\Leftrightarrow x-\frac{\pi}{4}=\pm\frac{\pi}{2}$$

$$\Leftrightarrow x=\frac{\pi}{4}\pm\frac{\pi}{2}=\frac{3\pi}{4}\text{ or }-\frac{\pi}{4}$$

c. Carefully construct a graph of the function showing how it passes through the points where y = –1, y = 0, y = 1 and how it approaches the vertical asymptotes. SOLN:



8. Suppose *sin t* = 8/17 and *t* is in the first quadrant. Find the following:

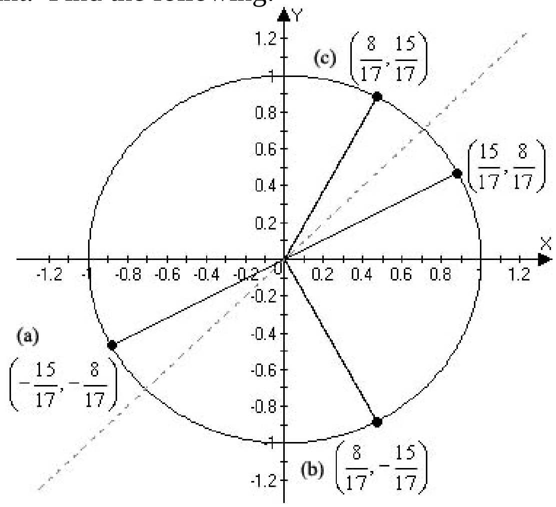
a. $\sin(t-\pi) = -\dfrac{8}{17}$

b.

$$\sin\left(t-\frac{\pi}{2}\right) = -\cos t = -\sqrt{1-\left(\frac{8}{17}\right)^2}$$

$$= -\sqrt{1-\frac{64}{289}} = -\sqrt{\frac{225}{289}} = -\frac{15}{17}$$

c. $\sin\left(\dfrac{\pi}{2}-t\right) = \dfrac{15}{17}$

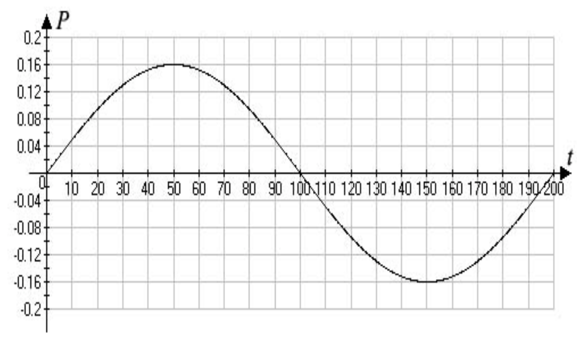


9. The graph to the right shows the air pressure, *P*, (in atm.) as a function of time (in milliseconds).

a. Find the amplitude and period of the function.
SOLN: $a = 0.16$ and the period = 200

b. Give a formula for the function, as a sinusoid in *t*.
SOLN: $y = 0.16\sin(\pi t/100)$

c. What is the frequency of oscillation, in cycles per second?
SOLN: 1/200 = 0.005 oscillations per milliseconds = 5 Hz.



Complete the table of values for $f(t) = 2\sin(\pi t) + \sin(3\pi t)$, plot the points and sketch a graph.

t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$2\sin(\pi t)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0
$\sin(3\pi t)$	0	1	$\frac{\sqrt{2}}{2}$	0	-1	0	$\frac{\sqrt{2}}{2}$	1	0
$f(t)$	0	2	$\frac{3\sqrt{2}}{2}$	$\sqrt{3}$	1	$\sqrt{3}$	$\frac{3\sqrt{2}}{2}$	2	0