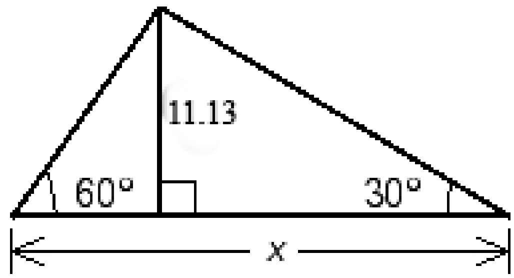
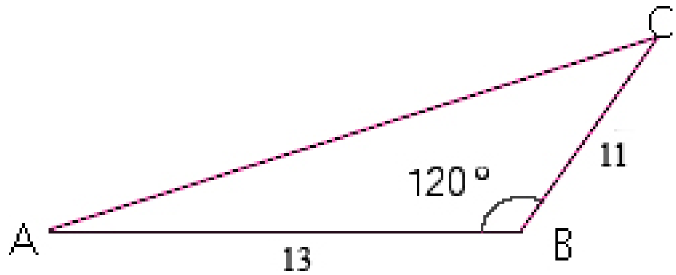
Math 5 – Trigonometry – Chapter 4 Test – Fair Game

1. The angle of elevation to the top of the Diesel Mechanics building from a point 82 feet from the base is 0.5 radians. Approximate the height the Diesel Mechanics building to the nearest foot.

2. Find x correct to four significant digits:

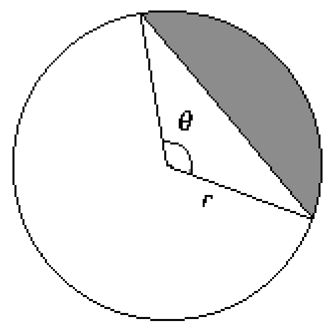


3. Find the area of the triangle at right:

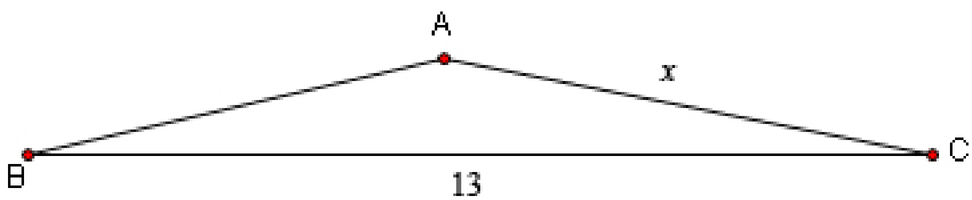


4. If $\theta = \frac{2\pi}{3}$ find the following. Approximate to four significant digits if necessary.
   a. $\tan^2(\theta)$
   b. $\tan(\theta^2)$

5. Find the area of the shaded region in the figure if $r = 13$ and $\theta = 110°$.

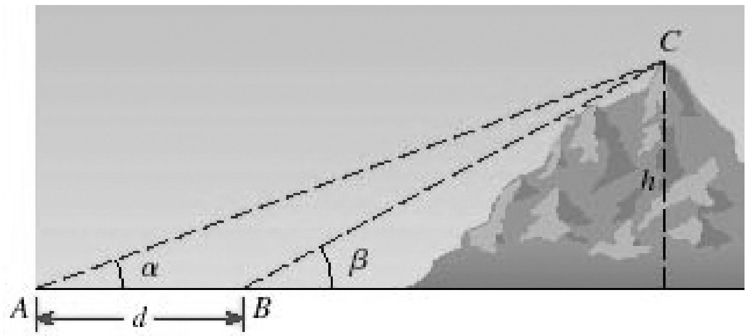


6. Use the law of sines to find the value of x in the diagram below. Assume that $\angle ABC = 13^\circ$ and $\angle ACB = 11^\circ$

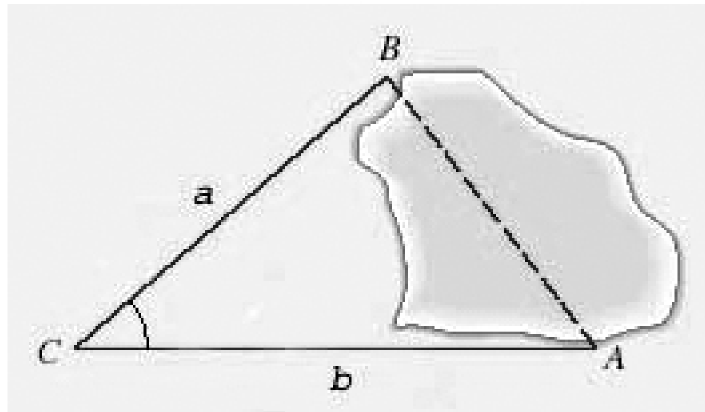


7. Sketch the triangle with $\angle A = 32^\circ$, $\angle C = 68^\circ$ and $b = 13.11$, then solve the triangle.

8. To calculate the height of a mountain, angles $\alpha = 11°$, $\beta = 13°$ and $d = 311$ ft are measured. Use the formula $h = d\dfrac{\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ to calculate the height.



9. To find the distance across a lake, a surveyor has taken the measurements $a = 11$ mi., $b = 13$ mi. and $\angle C = 40^\circ$. Find the distance across the lake using this information. Round to 2 significant digits.



10. A toy bicycle with one wheel of diameter 11cm and a bigger wheel with diameter 13cm is rolling along so that the big wheel is rolling at 10 rotations per minute.
    a. What is the angular speed of the little wheel?
    b. What is the linear speed of the bicycle?

11. Suppose we have vectors and $\vec{v} = 11\hat{i} - 13\hat{j}$
    a. Draw and label these vectors together in the *x*-*y* plane, assuming each has its initial point at (0,0).
    b. Find the angle between these two vectors.
    c. Find the length of $\vec{u}$ and the length of $\vec{v}$.
    d. Find the lengths of $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$

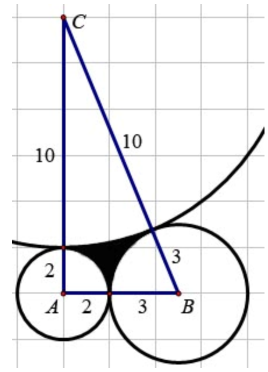
12. Suppose $\vec{v} = 11\hat{i} - 13\hat{j}$. Find a value of b so that the vector $\vec{u} = 10\hat{i} + b\hat{j}$ is orthogonal to *v*.

13. What is the central angle subtended by an arc length of 3π in a circle of radius 15? Write the radian measure of the angle.

14. Find the arc length in a circle of radius 4 that is subtended by an angle with radian measure 3.

15. What is the radius of a circle where a sector with central angle of 72° has area = π?

16. Three circles with radii 2cm, 3cm and 10cm are externally tangent to one another, as shown in the figure at right.



   a. Show that triangle ABC is a right triangle.

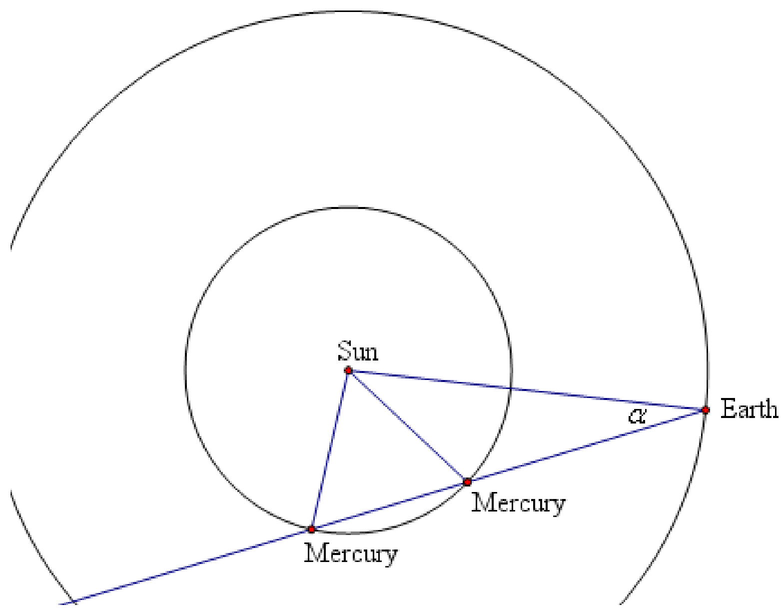
   b. Approximate to the nearest hundredth of a degree measures for $\angle B$ and $\angle C$, interior to triangle ABC.

   c. Approximate the area of the shaded region between the three circles to the nearest hundredth of a square cm.

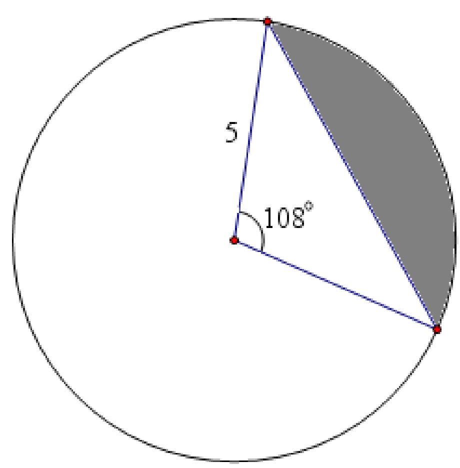
17. A bicycle with wheels of radius 0.3 meters is rolling at a linear speed of 114 meters per minute.
   a. What is the angular speed of the wheel is rad/min?
   b. How many times do the wheels rotate in 10 minutes?

18. The elongation α for Mercury is the angle formed by the planet, Earth and Sun, as shown in the diagram at right. Assume the distance from Mercury to the sun is 0.387 AU (38.7% of the distance from Earth to Sun) and that $\alpha = 18^\circ$. Since this is an ASS situation, there are two triangles which satisfy these conditions, as shown at right.

   Find both possible distances from Earth to Mercury.



19. Find the area of the shaded region in the figure at right.



20. Approximate to the nearest hundredth of a degree, the interior angles of a triangle with sides 4, 5 and 6.

21. Suppose an interior angle of a triangle measures 1 rad. and is nested between sides of lengths 11 and 14. What is the length of the side opposite that angle?

22. Suppose we have vectors $\vec{u} = \langle 1, 4 \rangle$ and $\vec{v} = 2\hat{i} - 3\hat{j}$
   a. Draw and label these vectors together in the *x*-*y* plane, assuming each has its initial point at (0,0).
   b. Find the length of $\vec{u}$ and the length of $\vec{v}$.
   c. Find the lengths of $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$
   d. Find the angle between these two vectors.
      Use the formula $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

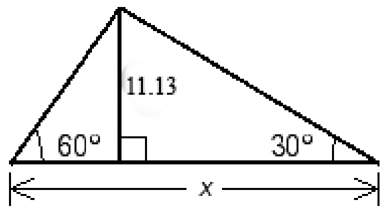
Math 5 – Trigonometry – Chapter 4 Test Fair Game Solutions

1. The angle of elevation to the top of the Diesel Mechanics building from a point 82 feet from the base is 0.5 radians. Approximate the height the Diesel Mechanics building to the nearest foot.

   SOLN: $\tan(0.5) = \frac{h}{82} \Rightarrow h \approx 0.5463(82) \approx 45$ ft.

2. Find x correct to four significant digits.

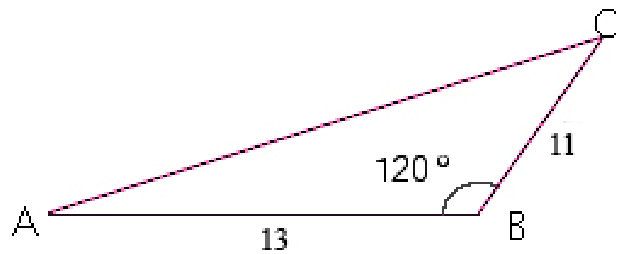
   SOLN: $x = \frac{11.13}{\sqrt{3}} + 11.13\sqrt{3} \approx 25.70$



3. Find the area of the triangle at right:

   SOLN: A = 11(13)sin(120°)/2 =

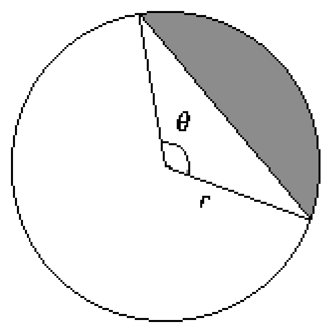
   $= \frac{143\sqrt{3}}{4} \approx 61.92$



4. If $\theta = \frac{2\pi}{3}$ find the following.

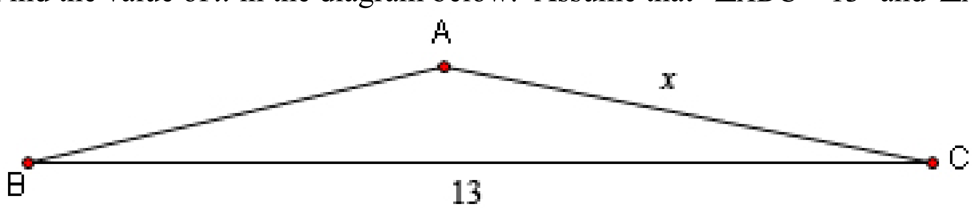
   e. $\tan^2(\theta) = \tan^2\left(\frac{2\pi}{3}\right) = \left(\sqrt{3}\right)^2 = 3$

   f. $\tan\left(\theta^2\right) = \tan\left(\frac{4\pi^2}{9}\right) \approx \tan 4.3865 \approx 2.959$



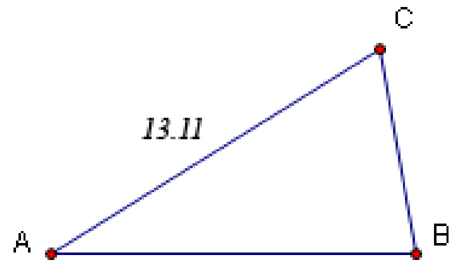
5. Find the area of the shaded region in the figure if $r = 13$ and $\theta = 110°$.
   SOLN:
   Area of sector – area of triangle =
   $\frac{r^2\theta}{2} - \frac{1}{2}r^2\sin\theta = \frac{13^2 11\pi}{2(18)} - \frac{13^2}{2}\sin\left(\frac{11\pi}{18}\right) = \frac{1859\pi}{36} - \frac{169}{2}\sin\left(\frac{11\pi}{18}\right) \approx 162.2 - 79.4 \approx 82.8$

6. Find the value of x in the diagram below. Assume that $\angle ABC = 13^\circ$ and $\angle ACB = 11^\circ$



   SOLN: $\frac{x}{\sin 13^\circ} = \frac{13}{\sin 156^\circ} \Leftrightarrow x = \frac{13\sin 13^\circ}{\sin 156^\circ} \approx \frac{13(0.22495)}{(0.40674)} \approx 7.190$

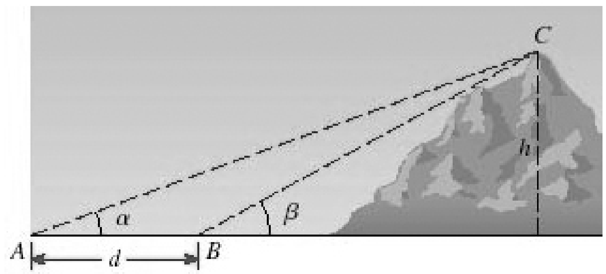
7. Sketch the triangle with $\angle A = 32^\circ$, $\angle C = 68^\circ$ and $b = 13.11$, then solve the triangle.



SOLN: $a = \dfrac{13.11\sin 32^\circ}{\sin 80^\circ} \approx 7.054$, $\angle b = 80^\circ$ and

$c = \dfrac{13.11\sin 68^\circ}{\sin 80^\circ} \approx 12.34$

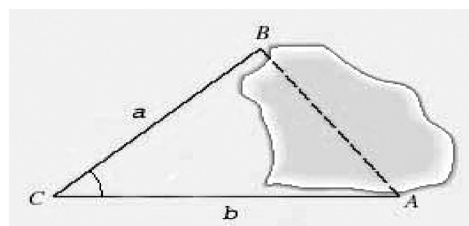
8. To calculate the height of a mountain, angles $\alpha = 11°$, $\beta = 13°$ and $d = 311$ ft are measured. Use the formula $h = d\dfrac{\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ to calculate the height.



SOLN: $h = d\dfrac{\sin\alpha\sin\beta}{\sin(\beta-\alpha)} = \dfrac{311\sin 11^\circ\sin 13^\circ}{\sin 2^\circ} \approx \dfrac{311(0.19081)(0.22495)}{0.03490} \approx 382.5$ ft

(Not much of a mountain.)

9. To find the distance across a lake, a surveyor has taken the measurements $a = 11$ mi., $b = 13$ mi. and $\angle C = 40^\circ$. Find the distance across the lake using this information. Round to 2 significant digits.



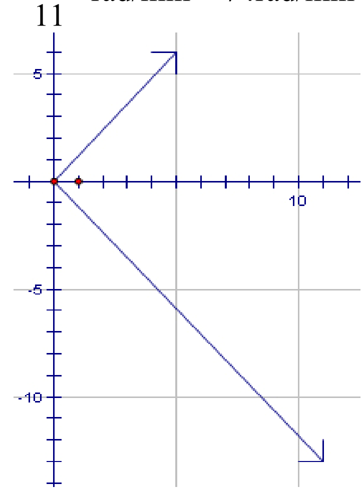
SOLN: By the law of cosines, the square of the distance is
$11^2 + 13^2 - 2(11)(13)\cos(40^\circ) \approx 290 - 286(0.76604) \approx 70.91$ So $AB \approx 8.421$

10. A toy bicycle with one wheel of diameter 11cm and a bigger wheel with diameter 13cm is rolling along so that the big wheel is rolling at 10 rotations per minute. What is the angular speed of the little wheel?

SOLN: First find the linear speed of the bike: $v = \omega r = \dfrac{10\text{rotations}}{\text{min}} \times \dfrac{2\pi}{\text{rotation}} \times 6.5\text{cm} = 130\pi\dfrac{\text{cm}}{\text{min}}$

Then find the angular velocity of the little wheel: $\omega = \dfrac{v}{r} = \dfrac{130\pi\text{cm/min}}{5.5\text{cm}} = \dfrac{260\pi}{11}\text{rad/min} \approx 74\text{rad/min}$

11. Suppose we have vectors $\vec{u} = 5\hat{i} + 6\hat{j}$ and $\vec{v} = 11\hat{i} - 13\hat{j}$



a. Draw and label these vectors together in the *x-y* plane, assuming each has its initial point at (0,0). (SOLN: See diagram at right.)

b. Find the angle between these two vectors.
SOLN: First find the lengths of the vectors:
$|\langle 11,-13\rangle| = \sqrt{11^2 + 13^2} = \sqrt{290}$ and $|\langle 5,6\rangle| = \sqrt{61}$ so that
$\theta = \cos^{-1}\left(\dfrac{\langle 11,-13\rangle\cdot\langle 5,6\rangle}{\sqrt{61}\sqrt{290}}\right) \approx \cos^{-1}\left(\dfrac{-23}{133}\right) \approx 100^\circ$

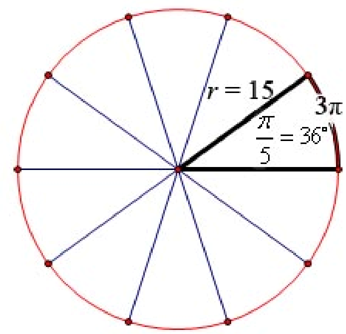
c. Find the length of $\vec{u}$ and the length of $\vec{v}$. (SOLN: See above.)

d. $|\vec{u} + \vec{v}| = |\langle 16,-7\rangle| = \sqrt{256 + 49} = \sqrt{305}$ and
$|\vec{u} - \vec{v}| = |\langle \vec{u} - \vec{v}\rangle| = \sqrt{397}$

12. Suppose $\vec{v} = 11\hat{i} - 13\hat{j}$. Find a value of b so that the vector $\vec{u} = 10\hat{i} + b\hat{j}$ is orthogonal to *v*.

SOLN: $\vec{v} \cdot \vec{u} = \langle 11, -13 \rangle \cdot \langle 10, b \rangle = 110 - 13b = 0 \Leftrightarrow b = \frac{110}{13}$

13. What is the central angle subtended by an arc length of 3π in a circle of radius 15? Write the radian measure of the angle.



SOLN: Using the formula $s = r\theta$, we have $3\pi = 15\theta \Rightarrow \boxed{\theta = \frac{\pi}{5}}$

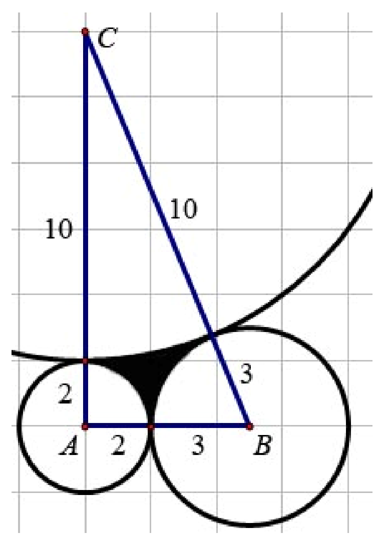
$\frac{\pi}{5} = 36^\circ$

14. Find the arc length in a circle of radius 4 that is subtended by an angle with radian measure 3.
SOLN: Same formula as in #1, but with different givens:
$s = r\theta = 4*3 = 12$.

15. What is the radius of a circle where a sector with central angle of 72° has area = π?

SOLN: Use the formula $A = r^2\theta/2$ with $\theta = 72\pi/180 = 2\pi/5$. That is $\pi = r^2\pi/5$ so $\boxed{r = \sqrt{5}}$.

16. Three circles with radii 2cm, 3cm and 10cm are externally tangent to one another, as shown in the figure at right.



a. Show that triangle ABC is a right triangle.
SOLN: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.

b. Approximate to the nearest hundredth of a degree measures for $\angle B$ and $\angle C$, interior to triangle *ABC*.
SOLN: $\angle B = \arcsin(12/13) \approx 67.38^\circ$,

$\angle C \approx 90^\circ - 67.38^\circ = 22.62^\circ$

c. Approximate the area of the shaded region between the three circles to the nearest hundredth of a square cm.

SOLN: The portion in circle A is simply ¼ its area: π. The area in circle B is approximately 67.38/360 of 9π $\approx 1.6845\pi$ and the part in circle C is approximately 22.62/360 of 100 π $\approx 6.283\pi$. So the total area is approximately $30 - 8.968\pi \sim 1.83$ cm^2.

17. A bicycle with wheels of radius 0.3 meters is rolling at a linear speed of 114 meters per minute.

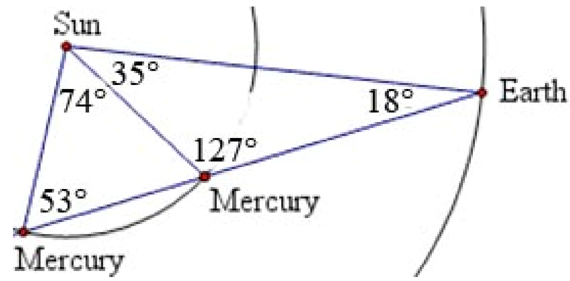
a. What is the angular speed of the wheel is rad/min?

SOLN: $\frac{114\text{meters}}{\text{min}} \times \frac{2\pi\text{ radians}}{2\pi(0.3)\text{meters}} = 380\frac{\text{rad}}{\text{min}}$

b. How many times do the wheels rotate in 10 minutes?

SOLN: $380\frac{\text{rad}}{\text{min}}\times\frac{1\,\text{revolution}}{2\pi\,\text{radians}}\times 10\,\text{min}=\frac{1900}{\pi}\approx 604.8$ revolutions in 10 minutes.

18. The elongation α for Mercury is the angle formed by the planet, Earth and Sun, as shown in the diagram at right. Assume the distance from Mercury to the sun is 0.387 AU (38.7% of the distance from Earth to Sun) and that α = 18°. Since this is an ASS situation, there are two triangles which satisfy these conditions, as shown at right. Find both possible distances from Earth to Mercury.

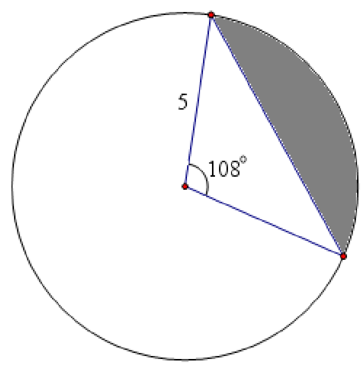


SOLN: Law of sines tells us that $\frac{\sin 18^\circ}{0.387}=\frac{\sin\angle M}{1}\Rightarrow\angle M=\sin^{-1}\left(\frac{\sin 18^\circ}{0.387}\right)\approx 53^\circ$ so the longer distance is $d\approx\frac{0.387}{\sin 18^\circ}\sin\left(180^\circ-53^\circ-18^\circ\right)=\frac{0.387}{\sin 18^\circ}\sin\left(109^\circ\right)\approx 1.184\,\text{AU}$ and the shorter distance is $d\approx\frac{0.387}{\sin 18^\circ}\sin\left(35^\circ\right)\approx 0.718\,\text{AU}$.

19. Find the area of the shaded region in the figure at right.

SOLN: Area = sector area – triangle area =

$\frac{108}{360}\pi 5^2-\frac{1}{2}*5*5\sin 108^\circ=\frac{15\pi}{2}-\frac{25}{2}\sin 72^\circ\approx 11.67$



20. Approximate to the nearest hundredth of a degree, the interior angles of a triangle with sides 4, 5 and 6.

SOLN: By the law of cosines, the angle opposite the longest side is $\theta=\cos^{-1}\left(\frac{6^2-5^2-4^2}{-2*4*5}\right)\approx 82.82^\circ$

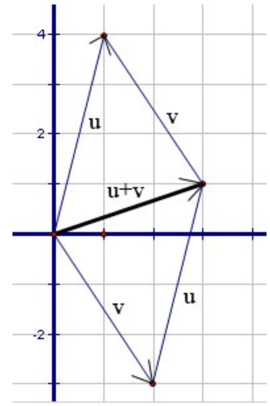
By law of sines, the angle opp. 5 = $\sin^{-1}\left(\frac{5\sin 82.82^\circ}{6}\right)\approx 55.77^\circ$ leaving 41.41° for the small angle.

21. Suppose an interior angle of a triangle measures 1 rad. and is nested between sides of lengths 11 and 14. What is the length of the side opposite that angle?

SOLN: By the law of cosines this length is

$\sqrt{11^2+14^2-2(11)(14)\cos 1}=\sqrt{121+196-308\cos\frac{180^\circ}{\pi}}\approx 12.27$

22. Suppose we have vectors $\vec{u} = \langle 1,4 \rangle$ and $\vec{v} = 2\hat{i} - 3\hat{j}$



a. Draw and label these vectors together in the *x*-*y* plane, assuming each has its initial point at (0,0). SOLN: see diagram at right.
b. Find the length of $\vec{u}$ and the length of $\vec{v}$.
   SOLN: $|\vec{u}| = \sqrt{1^2 + 4^2} = \sqrt{17}$ and $|\vec{v}| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$
c. Find the lengths of $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$
   SOLN: $|\vec{u} + \vec{v}| = |\langle 3,1 \rangle| = \sqrt{3^2 + 1^2} = \sqrt{10}$ and
   $|\vec{u} - \vec{v}| = |\langle -1,7 \rangle| = \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$
d. Find the angle between these two vectors. Use the formula
   $\theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \cos^{-1}\left(\frac{2-12}{\sqrt{17 \cdot 13}}\right) \approx 132.27^\circ$