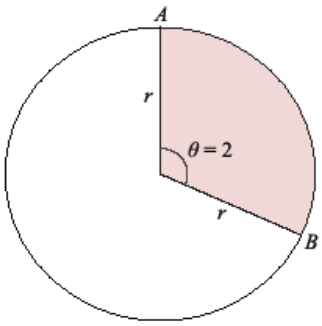
Math 5 – Trigonometry – Chapter 4 Test – fall '10 Name_____
Show your work for credit. Write all responses on separate paper.

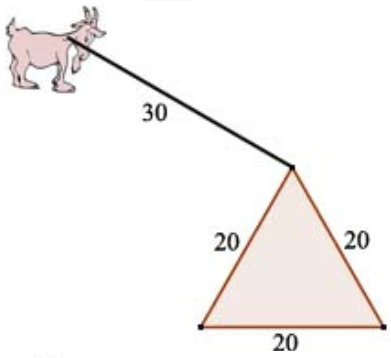
1. In the circle at right, arc length *AB* subtends a central angle of 2 radians.



   a. Suppose *AB* is 300 meters.
   What is the radius?

   b. Suppose the area of sector *AOB* is 5 square millimeters.
   What is the radius?

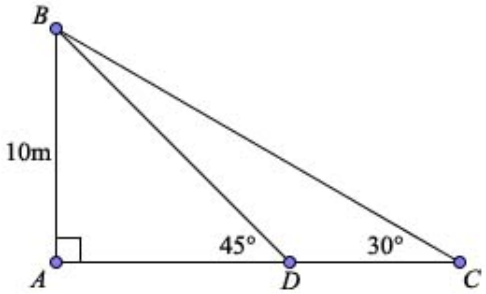
2. A goat is tethered by a 30 foot rope to a corner of a structure in the shape of an equilateral triangle with side length 20 feet, as seen from above in the figure at right. Neither the goat nor the tether can enter the structure.



   a. What is grazing area available to the goat?

   b. What is the outer perimeter of the goat's grazing area?

3. Given that *AB* = 10 meters in the diagram at right, find the length of *CD*.



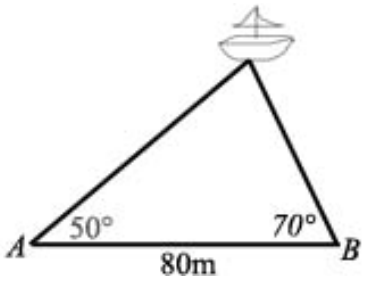
4. A 7 inch diameter disc rotates at 45 revolutions per minute.

   a. Find the angular speed of the disc, in degrees per second.

   b. What is the linear speed of the rim of the disc,
   in feet per second?

5. Phoebe is the outermost moon of Saturn and revolves at a distance of 23.5×10^6 km from Saturn once every 550 days. Find the linear speed of Phoebe relative to Saturn in meters per second.

6. Observers of a ship from the shoreline *AB* are located at *A* and *B* (see diagram at right) and measure the ship to shore angles of the ship from point 80 meters apart. How far from shore is the ship?



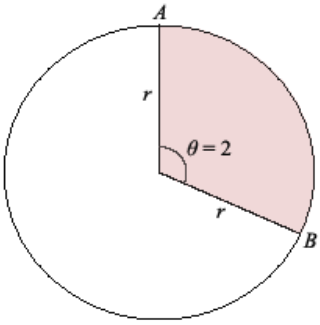
7. In ΔABC, $A = 72°$, $b = 10.0$ meters and $a = 9.7$ meters.
   a. Find two different possible values for *c*.
   b. Find two different possible areas for ΔABC.

8. In ΔABC, $a = 11$, $b = 15$ and $c = 10$.
   a. Approximate to the nearest hundredth the radian measures of the interior angles of the triangle.
   b. Find the area of the triangle in simplest radical form.

9. The minute hand of a certain clock has length 6 inches and the hour hand has length 4 inches.
   a. What angle to the hands of the clock form at 1:30 ?
   b. What is the distance between the tips of the hands at 1:30 ?

Math 5 – Trigonometry – Chapter 4 Test Solutions – fall '10

1. In the circle at right, arc length *AB* subtends a central angle of 2 rads.



   a. Suppose *AB* is 300 meters. What is the radius?

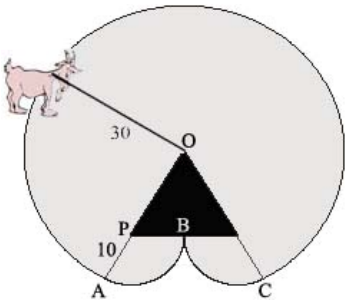
      SOLN: Using the definition of radian measure, $\theta = \frac{s}{r}$ we have

      $2 = \frac{300}{r} \Leftrightarrow r = 150$ meters.

   b. If the area of sector *AOB* is 5 square millimeters then, since the area of the sector is proportional to the central angle,

      $\frac{\theta}{2\pi} = \frac{A}{\pi r^2} \Leftrightarrow \frac{2}{2\pi} = \frac{5}{\pi r^2} \Leftrightarrow r^2 = 5$, So $r = \sqrt{5}$ mm

2. A goat is tethered by a 30 foot rope to a corner of a equilateral triangular fence with side length 20 feet.



   a. What is grazing area available to the goat, provided there are no other restrictions?

      SOLN: There are three sectors available to the goat, the big one with radius 30 and central angle $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ and the two little ones with radius 10 and central angle $\frac{2\pi}{3}$. Thus the area available

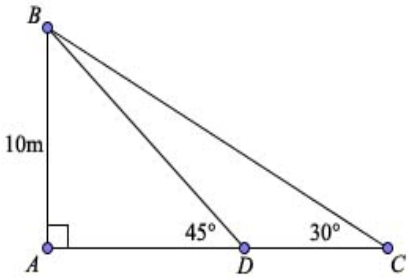
      is $\frac{5\pi}{3}\frac{30^2}{2} + 2\left(\frac{2\pi}{3}\frac{10^2}{2}\right) = 750\pi + \frac{200\pi}{3} = \frac{2450\pi}{3} \approx 2565.6 \text{ ft}^2$.

   b. What is the outer perimeter of the goat's grazing area?

      SOLN: There are three arcs: the big one with radius 30 and central angle $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ and the two little ones with radius 10 and central angle $\frac{2\pi}{3}$. So the outer perimeter is

      $30\left(\frac{5\pi}{3}\right) + 2(10)\left(\frac{2\pi}{3}\right) = 50\pi + \frac{40\pi}{3} = \frac{190\pi}{3} \approx 199.0$ ft

3. Given that *AB* = 10 meters in the diagram at right, find the length of *CD*



   SOLN: $CD = AC - AD = 10\sqrt{3} - 10 = 10\left(\sqrt{3} - 1\right) \approx 7.32$ meters

4. A disc 7 inches in diameter rotates at a rate of 45 revolutions per minute.

   a. What is the angular speed of the disc, in degrees per second?

      SOLN: $\frac{45 \text{ rotations}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rotation}} = \frac{90\pi}{\text{min}} \times \frac{180^\circ}{\pi} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{270^\circ}{\text{sec}}$

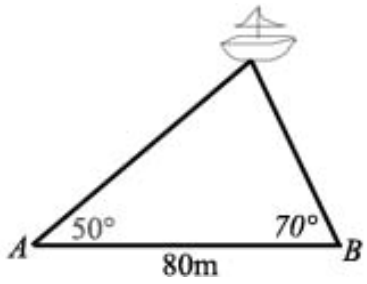
   b. What is the linear speed of the rim of the disc, in feet per second?

      SOLN: $90\pi \frac{\text{rad}}{\text{min}}\left(\frac{7}{2}\text{inch}\right) = 315\pi \frac{\text{inches}}{\text{min}} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{7\pi}{16} \text{ fps} = 1.374 \text{ fps}$

5. Phoebe is the outermost moon of Saturn and revolves at a distance of 23.5×10^6 km from Saturn once every 550 days. Find the linear speed of Phoebe relative to Saturn in meters per second.

$$v = \omega r = \frac{1 \text{ rotation}}{550 \text{ days}} \times \frac{2\pi \text{ rad}}{\text{rotation}} \times 23.5 \times 10^6 \text{ km} = \frac{23.5\pi \times 10^6 \text{ km}}{275 \text{ days}} \approx 267000 \frac{\text{km}}{\text{day}} \times \frac{1 \text{day}}{24 \text{hr}} \times \frac{1 \text{hr}}{3600 \text{ sec}} \approx 3100 \text{m/s}$$

6. Observers of a ship from the shoreline AB are located at A and B (see diagram at right) and measure the ship to shore angles of the ship from point 80 meters apart. How far from shore is the ship?



SOLN: By the law of sines, $\frac{b}{\sin 70^\circ} = \frac{80}{\sin 60^\circ} \Leftrightarrow b = \frac{160 \sin 70^\circ}{\sqrt{3}}$

so that the ship to shore distance is $\frac{160 \sin 70^\circ}{\sqrt{3}} \sin 50^\circ \approx 66.5 \text{ m}$

7. In ΔABC, $A = 72^\circ$, $a = 10.0$ meters and $b = 9.1$ meters.
   a. Find c. (Note that since $a > b$, there is only one solution.) SOLN: By the law of sines,

   $$\frac{10}{\sin 72^\circ} = \frac{9.1}{\sin B} \Leftrightarrow \sin B = \frac{9.1 \sin 72^\circ}{10} \approx 0.8655 \Rightarrow B = \sin^{-1} 0.8655 \approx 59.9^\circ$$

   This means that $C = 180^\circ - A - B \approx 48.1^\circ$ and so $\frac{10}{\sin 72^\circ} = \frac{c}{\sin C} \Leftrightarrow c \approx \frac{10 \sin 48.1}{\sin 72^\circ} \approx 7.82 \text{ m}$

   b. Find the area for ΔABC. SOLN: $\frac{1}{2} ab \sin C = 45.5 \sin(48.1) \approx 39.4 \text{ m}^2$.

7' In ΔABC, $A = 72^\circ$, $a = 9.7$ meters and $b = 10$ meters.
   a. Find both values of c. (Note that since $b \sin 72^\circ < a < b$, there are two solutions.)
   SOLN: By the law of sines,

   $$\frac{9.7}{\sin 72^\circ} = \frac{10}{\sin B} \Leftrightarrow \sin B = \frac{10 \sin 72^\circ}{9.7} \approx 0.9805 \Rightarrow B = \sin^{-1} 0.9805 \approx 78.66^\circ \approx 1.37$$

   or $B' = 180^\circ - 78.66^\circ = 101.34^\circ \approx 1.77$, whence $C = 29.34^\circ \approx 0.512$ or $C' = 6.66^\circ \approx 0.116$

   So $\frac{9.7}{\sin 72^\circ} = \frac{c}{\sin C} \Leftrightarrow c \approx \frac{9.7 \sin 29.34^\circ}{\sin 72^\circ} \approx 5.00 \text{ m}$ whereas $\frac{9.7}{\sin 72^\circ} = \frac{c'}{\sin C'} \Leftrightarrow c' \approx \frac{9.7 \sin 6.66^\circ}{\sin 72^\circ} \approx 1.18$

   c. Two areas of ΔABC are $\frac{1}{2} ab \sin C = 48.5 \sin(48.1) \approx 23.8 \text{ m}^2$ or $\frac{1}{2} ab \sin C' = 48.5 \sin(6.66) \approx 5.62$

8. In ΔABC, $a = 11$, $b = 15$ and $c = 10$.
   a. Approximate to the nearest hundredth the radian measures of the interior angles of the triangle.
   SOLN: You can get any of the angles first using the law of cosines:

   $$\cos C = \frac{11^2 + 15^2 - 10^2}{2(11)(15)} = \frac{41}{55} \Leftarrow C = \cos^{-1}\left(\frac{41}{55}\right) \approx 41.80^\circ \approx 0.73 \text{ or}$$

   $$\cos B = \frac{11^2 + 10^2 - 15^2}{2(11)(10)} = \frac{-1}{55} \Leftarrow B = \cos^{-1}\left(\frac{-1}{55}\right) \approx 91.04^\circ \approx 1.59 \text{ or}$$

   $$\cos A = \frac{10^2 + 15^2 - 11^2}{2(10)(15)} = \frac{17}{25} \Leftarrow A = \cos^{-1}\left(\frac{17}{25}\right) \approx 47.2^\circ \approx 0.82$$ - These 3 angles sum to 3.14

   Then you can use the law of sines to find a second angle and the theorem for the sum of the interior angles of any triangles will most easily yield the third.
   b. Find the area of the triangle in simplest radical form.
   SOLN: The semiperimeter is (11+15+10)/2 = 18 so Heron's formula yields
   $\sqrt{18(3)(7)(8)} = \sqrt{16(9)(21)} = 12\sqrt{21}$

9. The minute hand of a certain clock has length 6 inches and the hour hand has length 4 inches.
   a. What angle to the hands of the clock form at 1:30 ?
   SOLN: A little examination shows the angle is $135^\circ = 3\pi/4$
   b. What is the distance between the tips of the hands at 1:30 ?
   SOLN: Using the law of cosines we have

   $$d^2 = 4^2 + 6^2 - 2(4)(6)\cos\frac{3\pi}{4} = 52 + 24\sqrt{2} \Leftarrow d = \sqrt{52 + 24\sqrt{2}} \approx 9.27 \text{ inches.}$$