Math 5 – Trigonometry – fall '10 – Chapter 2 Test Name____________________

Show all work for credit and write all responses on separate paper. Don't use a calculator.

1. Consider the line passing through the origin (0,0) and the center of the circle described by $(x-3)^2+(y-4)^2=25$. NOTE: The general form for a circle centered at (*h*, *k*) with radius *r* is $(x-h)^2+(y-k)^2=r^2$
   a. Write a formula for the function that describes this line in slope-intercept form: $f(x)=mx+b$.

   b. Write a formula for the function for the line parallel to this line and passing through (0,8).

   c. Write a formula for the function of the line through (0,8) and perpendicular to this line.

2. Compute and simplify the average rate of change of $f(x)=2x^3$ over the given interval. Recall that this average rate of change is the slope of the secant line connecting [*a*, *f*(*a*)] with [*b*, *f*(*b*)].

   a. [0, *h*]

   b. [–*h*, *h*]

3. Consider the quadratic $f(x)=3x^2-6x+2$
   a. Express the quadratic function in standard (vertex) form: $y=a(x-h)^2+k$

   b. Find the coordinates of the *x*-intercepts.

   c. Express the quadratic function in factored form: $y=a(x-r_1)(x-r_2)$

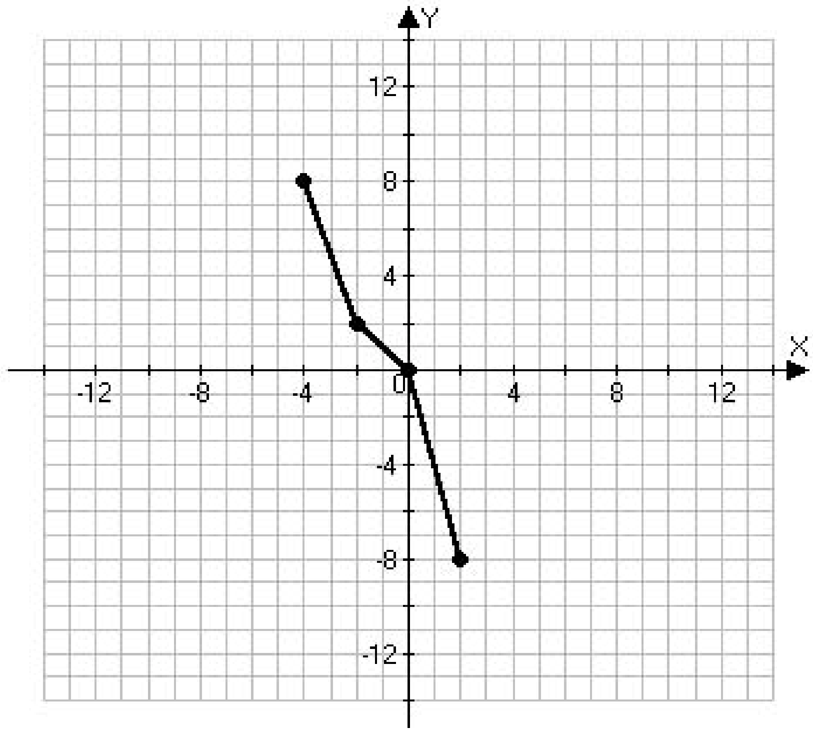
   d. Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.

4. Find the maximum value of the given function and state its range in interval notation.
   a. $f(x)=-4(x-1)^2+10$
   b. $f(x)=-2x^2+8x+1$

5. Consider the quadratic $f(x)=-2x^2+4x+3$
   a. Express the quadratic function in standard form.
   b. Sketch its graph showing the position of the vertex.
   c. What sequence of
      (i) vertical shift,
      (ii) reflection,
      (iii) vertical shrink, and
      (iv) horizontal shift
      would be required to transform this function to $y=x^2$?

6. Given the graph of $y = f(x)$ shown at right and the given transformation, tabulate the transformed coordinate values of the key points in the graph. Then plot the given transformation on the paper at right.

x	-4	-2	0	2
$f(x)$	8	2	0	-8



a. $y = 1 + f(x-2)$

x				
$1+f(x-2)$				

b. $y = \frac{1}{2} f(x/3)$

x				
$\frac{1}{2} f(x/3)$				

7. The surface area of a sphere is a function of the radius according to $S = f(r) = 4\pi r^2$ and the volumef a sphere is a function of the radius according to $V = g(r) = \frac{4}{3}\pi r^3$. Find a function that gives the surface area, *S*, as a function of the Volume, *V*.

8. Suppose $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-2}$.

a. What is the domain of f?
b. What is the range of f?
c. What is the domain of g ?
d. Find a formula for and determinethe domain of $(g \circ f)(x)$
e. Find a formula for and determine the domain of $(f \circ g)(x)$

9. Find a formula for the inverse function of $f(x) = \frac{1}{3}x - 2$ and sketch a graph for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y = x$.

Math 5 – Trigonometry – fall '10 – Chapter 2 Test Solutions

a. Consider the line passing through the origin (0,0) and the center of the circle described by $(x-3)^2+(y-4)^2=25$.

b. Write a formula for the function that describes this line in slope-intercept form: $f(x)=mx+b$.

SOLN: The line will pass through (0,0) and (3,4) so $m=\frac{4-0}{3-0}=\frac{4}{3}$ and since $b = 0$, $\boxed{y=\frac{4}{3}x}$

c. Write a formula for the function for the line parallel to this line and passing through (0,8).

SOLN: $b=8\Rightarrow \boxed{y=\frac{4}{3}x+8}$

d. Write a formula for the function of the line through (0,8) and perpendicular to this line.

SOLN: $m_\perp=-\frac{3}{4}$; $b=8\Rightarrow \boxed{y=-\frac{3}{4}x+8}$

2. Compute and simplify the average rate of change of $f(x)=2x^3$ over the given interval. Recall that this average rate of change is the slope of the secant line connecting [a, $f(a)$] with [b, $f(b)$].

a. [0, h] SOLN: $\frac{f(h)-f(0)}{h-0}=\frac{2h^3-0}{h}=2h^2$

b. [–h, h] SOLN: $\frac{f(h)-f(-h)}{h-(-h)}=\frac{2h^3-(-2h^3)}{2h}=\frac{4h^3}{2h}=2h^2$

3. Consider the quadratic $f(x)=3x^2-6x+2$

a. Express the quadratic function in standard (vertex) form: $y=a(x-h)^2+k$

SOLN: $f(x)=3x^2-6x+2=3(x^2-2x)+2=3(x^2-2x+2)+2-3=\boxed{3(x-1)^2-1}$

b. Find the coordinates of the x-intercepts.

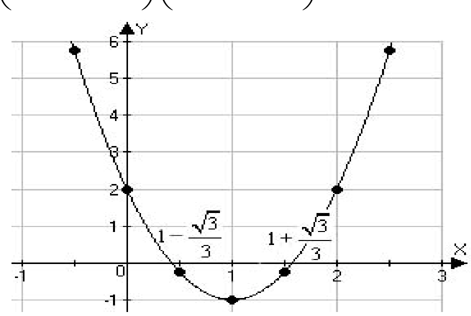
SOLN: $y=0\Leftrightarrow 3(x-1)^2-1=0\Leftrightarrow (x-1)^2=\frac{1}{3}\Leftrightarrow x-1=\frac{\pm\sqrt{3}}{3}\Leftrightarrow \boxed{x=1\pm\frac{\sqrt{3}}{3}}$

c. Express the quadratic function in factored form: $y=a(x-r_1)(x-r_2)$

SOLN: $y=3\left(x-\left(1-\frac{\sqrt{3}}{3}\right)\right)\left(x-\left(1+\frac{\sqrt{3}}{3}\right)\right)=3\left(x-1+\frac{\sqrt{3}}{3}\right)\left(x-1-\frac{\sqrt{3}}{3}\right)$

d. Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.
SOLN: A table of values is always helpful:

x	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
y	$\frac{23}{4}$	2	$-\frac{1}{4}$	−1	$-\frac{1}{4}$	2	$\frac{23}{4}$



4. Find the maximum value of the given function and state its range in interval notation.

a. $f(x) = -4(x-1)^2 + 10$

SOLN: The maximum value of occurs at the vertex where $y = 10$. The range is $[-\infty, 10)$

b. $f(x) = -2x^2 + 8x + 1$ SOLN: $f(x) = -2x^2 + 8x + 1$ $f(x) = -2(x-2)^2 + 9$

so the max value of occurs at the vertex where $y = 9$. The range is $[-\infty, 9)$

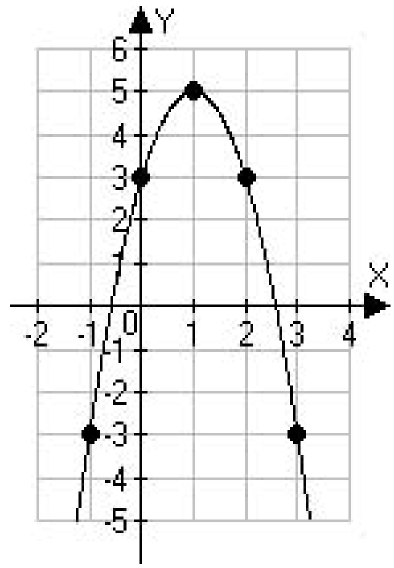
5. Consider the quadratic $f(x) = -2x^2 + 4x + 3$

a. Express the quadratic function in standard form.

SOLN: $f(x) = -2(x-1)^2 + 5 \Leftrightarrow y - 5 = -2(x-1)^2$

b. Sketch its graph showing the position of the vertex.

SOLN:



c. What sequence of

(i) vertical shift, SOLN: Shift down 5 by ($y \leftarrow y + 5$)

(ii) reflection, SOLN: Reflect across x-axis by ($y \leftarrow -y$)

(iii) vertical shrink, SOLN: Shrink vertically by ($y \leftarrow 2y$)

(iv) horizontal shift SOLN: Shift left 5 by ($x \leftarrow x + 1$)

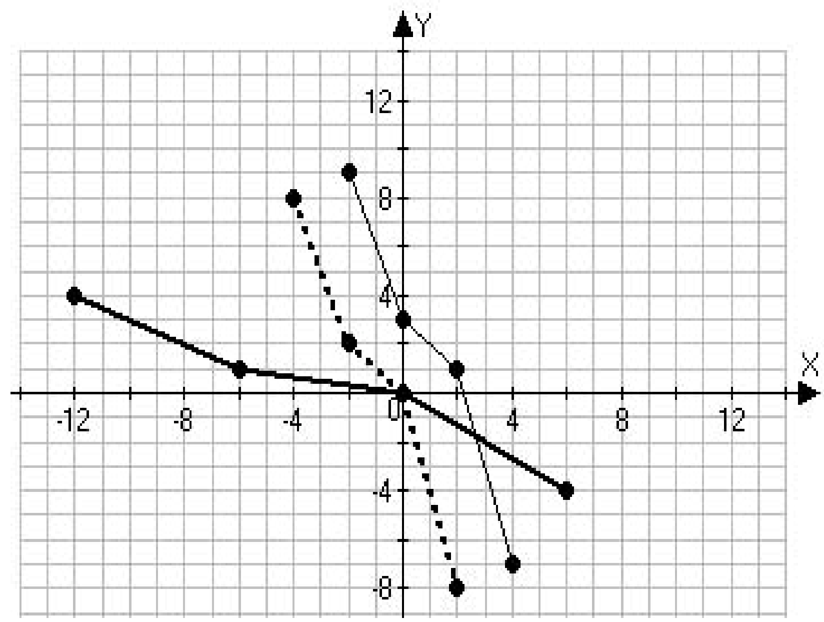
In the above order, these transforms lead to the following sequence of equations:

$\boxed{y - 5 = -2(x-1)^2} \rightarrow \boxed{y = -2(x-1)^2} \rightarrow \boxed{y = 2(x-1)^2}$

$\rightarrow \boxed{y = (x-1)^2} \rightarrow \boxed{y = x^2}$

6. Given the graph of $y = f(x)$ shown at right and the given transformation, tabulate the transformed coordinate values of the key points in the graph. Then plot the given transformation on the paper at right.

x	-4	-2	0	2
$f(x)$	8	2	0	-8



a. $y = 1 + f(x-2)$:

x	-2	0	2	4
$1 + f(x-2)$	9	3	1	-7

b. $y = \frac{1}{2} f(x/3)$:

x	-12	-6	0	6
$\frac{1}{2} f(x/3)$	4	1	0	-4

7. Find a function that gives the surface area, *S*, as a function of the Volume, *V*. SOLN: The surface area is $V = \frac{4}{3}\pi r^3 \Leftarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so $V = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3} = (4\pi)^{3/3}\left(\frac{9V^2}{16\pi^2}\right)^{1/3} = \left(36\pi V^2\right)^{1/3}$

8. Suppose $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-2}$.

   a. Then $(g \circ f)(x) = \frac{1}{\sqrt{x}-2}$ has domain $x \in [0,4) \cup (4,\infty)$

   b. And $(f \circ g)(x) = (x-2)^{-1/2}$ has domain $x \in (2,\infty)$

9. If $f(x) = \frac{1}{3}x - 2$ then $f^{-1}(x) = 3x + 6$.