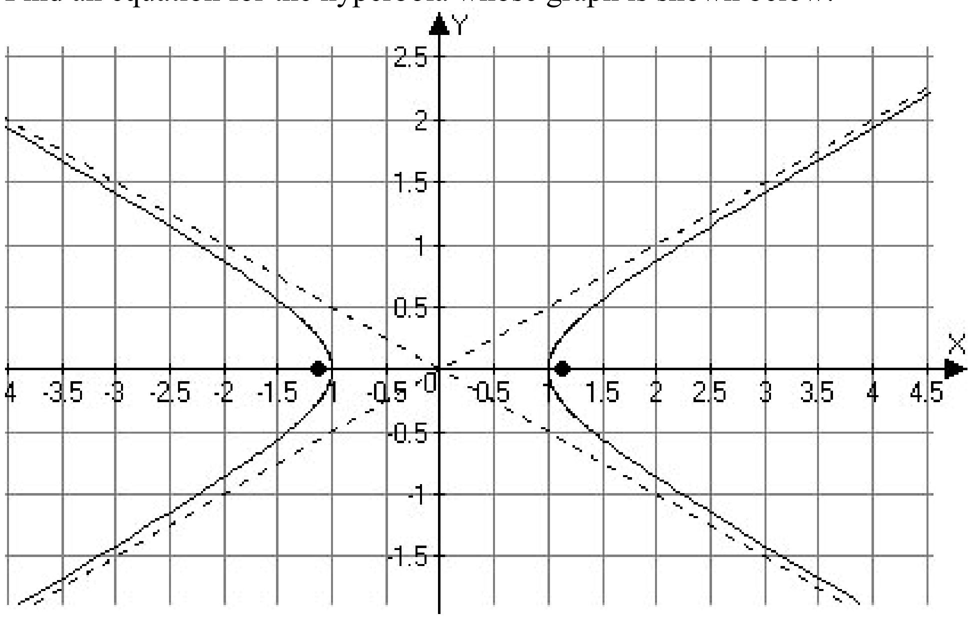
Math 5 – Trigonometry – fall '10 – Chapter 5 Test Name____________________________

Directions: Show all your work for credit. Write all responses on separate paper.

1. Consider the conic section described by $x^2 = 6(x - 2y)$.
   a. Find the coordinates of the vertex.
   b. Find the coordinates of the focus.
   c. Find an equation for the directrix.
   d. Find the focal diameter.
   e. Construct a graph showing these features.

2. Consider the ellipse centered at (12,13) with tangent lines along the coordinate axes.
   a. Where are the vertices? Give coordinates.
   b. Where are the foci? Give coordinates.
   c. Find the eccentricity.
   d. Write parametric equations for this conic.
   e. Construct a careful graph showing the key features.

3. Find an equation for the hyperbola whose graph is shown below:



4. Write a polar equation of a conic with the focus at the origin and the given data. Tabulate the *x*-intercepts and *y*-intercepts and sketch a graph for each.

   a. An ellipse with vertex at $(r, \theta) = (3, \pi)$ and eccentricity 0.25

   b. A hyperbola with vertex at $(r, \theta) = (1.2, \pi/2)$ and eccentricity 1.5

5. Consider the curve given by parametric equations
$$\boxed{\begin{array}{l} x = 3\sec(t) \\ y = 5 + 4\tan(t) \\ 0 \le t \le 2\pi \end{array}}$$

   a. Eliminate the parameter *t* to obtain an equation for this curve in rectangular coordinates.
   b. Construct a careful graph for the curve.

Math 5 – Trigonometry – fall '10 – Chapter 5 Test Solutions

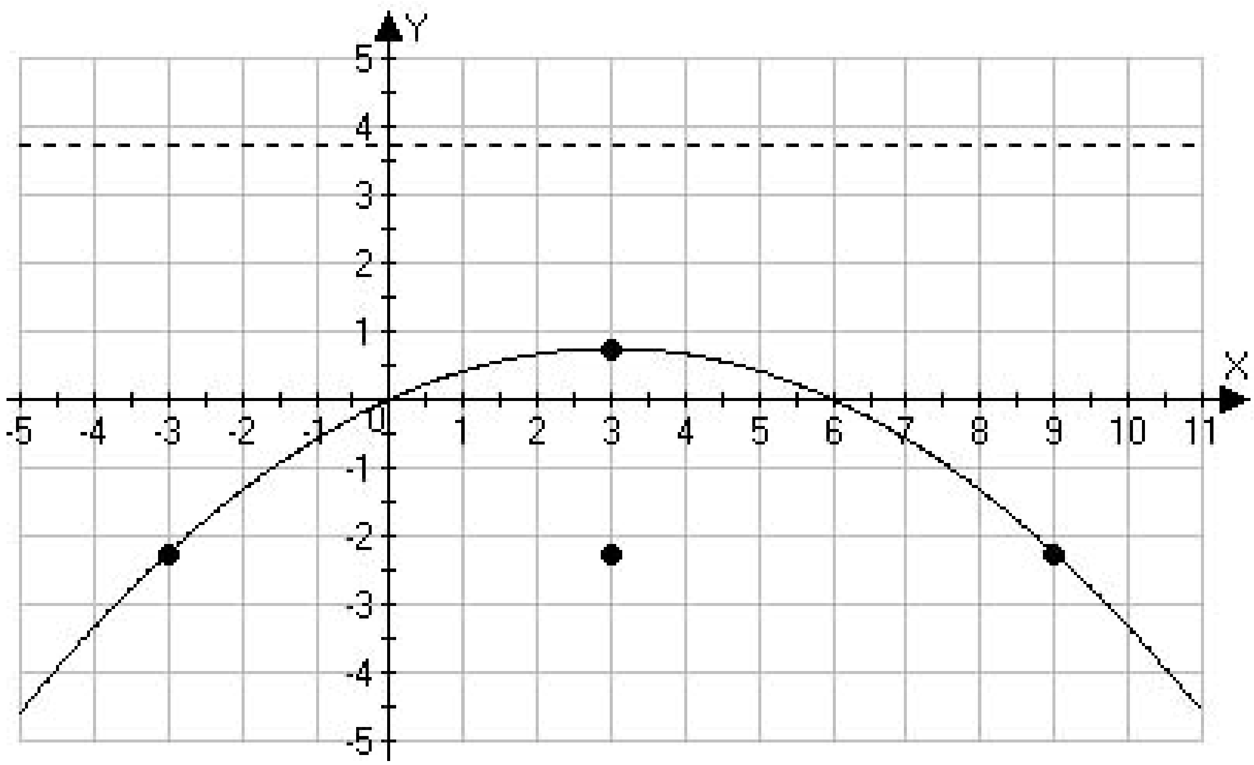
1. Consider the conic section described by $x^2 = 6(x - 2y)$.
   a. Find the coordinates of the vertex.
   SOLN: $x^2 = 6(x - 2y) \leftrightarrow x^2 - 6x = -12y \leftrightarrow x^2 - 6x + 9 = -12y + 9 \leftrightarrow (x-3)^2 = -12(y - \frac{3}{4})$
   so the vertex is at $(3, \frac{3}{4})$

   b. Find the coordinates of the focus.
   SOLN: $p = 3$ and the parabola opens downwards from its vertex,
   so the focus is at $\left(3, \frac{3}{4} - 3\right) = \left(3, -\frac{9}{4}\right)$

   c. Find an equation for the directrix.
   SOLN: The directrix is a horizontal line 3 units above the vertex: $y = \frac{15}{4}$

   d. Find the focal diameter.
   SOLN: The focal diameter has length 12 and extends from $\left(-3, -\frac{9}{4}\right)$ to $\left(6, -\frac{9}{4}\right)$.

   e. Construct a graph showing these features.
   SOLN:



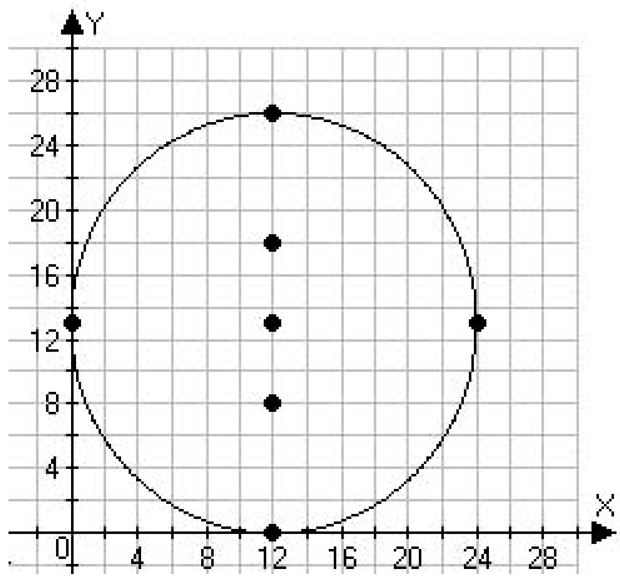
2. Consider the ellipse centered at (12,13) with tangent lines along the coordinate axes, $x = 0$ and $y = 0$.
   a. Where are the vertices? Give coordinates.
      SOLN: (12,0) and (12,26) are the major axis vertices and (0,13) and (24,13) are on the minor axis.

   b. Where are the foci? Give coordinates.
      SOLN: $c^2 = a^2 - b^2 = 13^2 - 12^2 = 169 - 144 = 25$, so $c = 5$ and foci at (12,13±5) = (12,8) and (12 18).

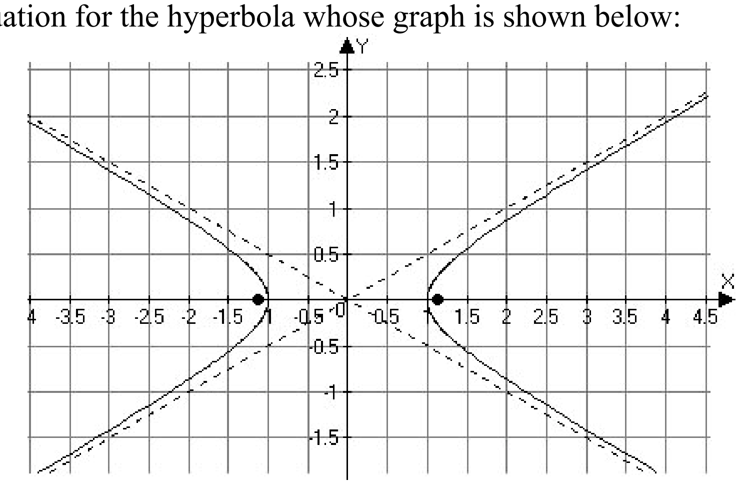
   c. Find the eccentricity.
      SOLN: eccentricity = c/a = 5/13.

   d. Write parametric equations for this conic.
      SOLN: $x = 12 + 12\cos(t)$ and $y = 13 + 13\sin(t)$.

   e. Construct a careful graph showing the key features.



3. Find an equation for the hyperbola whose graph is shown below:



SOLN: Evidently, the center is at (0,0) and the vertices are at (±1,0) and the slopes of the asymptotes are ±½ so $a = 1$ and $b/a = ½$. Thus $b = ½$. Combining this information with the formula $c^2 = a^2 + b^2 = 5/4$
The equation of the for the hyperbola is then $x^2 - 4y^2 = 1$

4. Write a polar equation of a conic with the focus at the origin and the given data. Tabulate the x-intercepts and y-intercepts and sketch a graph for each.

   a. An ellipse with vertex at $(r, \theta) = (3, \pi)$ and eccentricity 0.25
   There are two good solutions to this problem:
   SOLN1: With a focus is at (0,0), the ellipse could open to the right from the vertex at $(x, y) = (-3, 0)$ and so the formula would be of the type $r = \dfrac{ed}{1 - e\cos\theta} = \dfrac{d/4}{1 - \dfrac{\cos\theta}{4}} = \dfrac{d}{4 - \cos\theta}$.

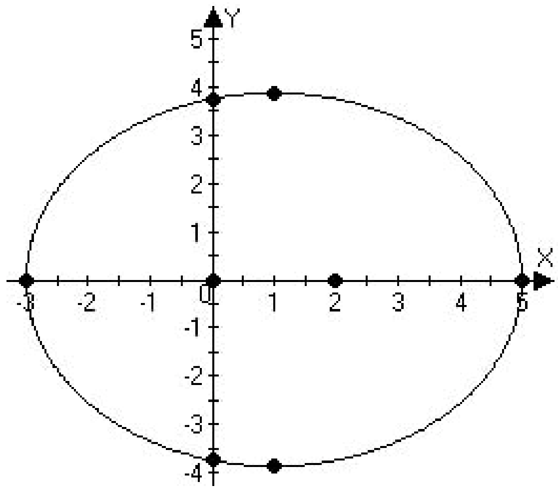
   At $\theta = \pi$, $3 = \dfrac{d}{4 - \cos\pi} = \dfrac{d}{5} \Rightarrow d = 15$ so the polar equation for ellipse is $\boxed{r = \dfrac{15}{4 - \cos\theta}}$.

   The intercepts are

r	5	15/4	3	15/4
θ	0	$\pi/2$	π	$3\pi/2$

$\Leftrightarrow$

x	5	0	-3	0
y	0	15/4	0	15/4



   SOLN2: …or the ellipse could open to the left and still have a vertex at $(x, y) = (-3, 0)$. In this case the formula is of the type $r = \dfrac{ed}{1 + e\cos\theta} = \dfrac{d/4}{1 + \dfrac{\cos\theta}{4}} = \dfrac{d}{4 + \cos\theta}$.

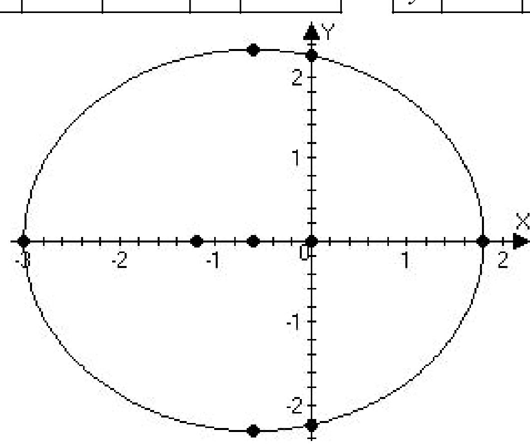
   At $\theta = \pi$, $3 = \dfrac{d}{4 + \cos\pi} = \dfrac{d}{3} \Rightarrow d = 9$ so the polar equation for ellipse is $\boxed{r = \dfrac{9}{4 + \cos\theta}}$.

   The intercepts are

r	9/5	9/4	3	9/4
θ	0	$\pi/2$	π	$3\pi/2$

$\Leftrightarrow$

x	9/5	0	-3	0
y	0	9/4	0	9/4



b. A hyperbola with vertex at $(r, \theta) = (1.2, \pi/2)$ and eccentricity 1.5

SOLN: Here the vertex is at $(x,y) = (0,6/5)$ and so that branch of the hyperbola opens downward and the formula is of the type $r = \frac{ed}{1+e\sin\theta} = \frac{3d/2}{1+\frac{3\sin\theta}{2}} = \frac{3d}{2+3\sin\theta}$.

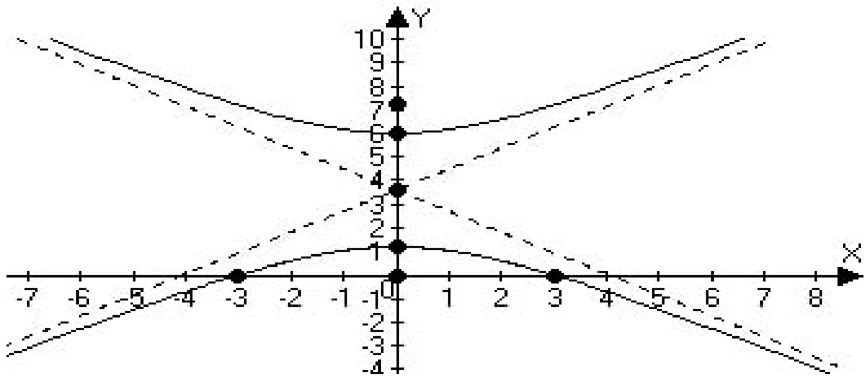
At $\theta = \pi/2$, $\frac{6}{5} = \frac{3d}{2+3\sin\frac{\pi}{2}} = \frac{3d}{5} \Rightarrow d = 2$ so the equation is $\boxed{r = \frac{6}{2+3\sin\theta}}$.

The intercepts are

r	3	6/5	3	−6
θ	0	$\pi/2$	π	$3\pi/2$

$\Leftrightarrow$

x	3	0	−3	0
y	0	6/5	0	6



5. Consider the curve given by parametric equations $\boxed{\begin{array}{l} x = 3\sec(t) \\ y = 5 + 4\tan(t) \\ 0 \le t \le 2\pi \end{array}}$

a. Eliminate the parameter t to obtain an equation for this curve in rectangular coordinates.

SOLN: $\frac{x^2}{9} - \frac{(y-5)^2}{16} = 1$

b. Construct a careful graph for the curve.