Math 5 – Trigonometry – Chapter 3 – Fair Game Problems

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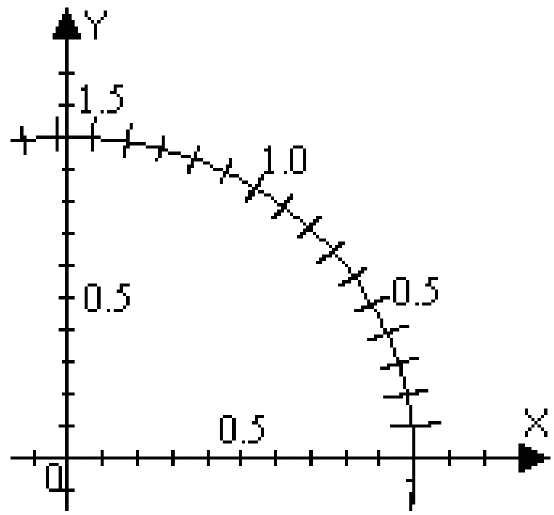
1. If the arclength $t = \frac{29\pi}{6}$ is traced counterclockwise along the unit circle from (1,0) then
   a. What is the reference number for t ?
   b. What are the coordinates of the terminal point $P(x,y)$?

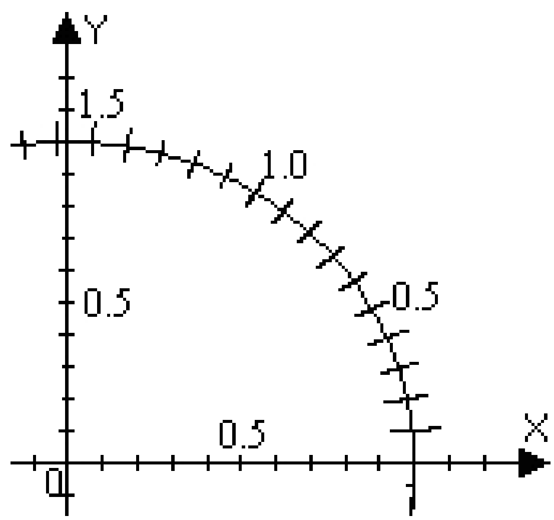
2. For arclength $t = \frac{31\pi}{6}$ extending counterclockwise along the unit circle from (1,0)
   a. Find the reference number for t.
   b. Find the coordinates of the terminal point $P(x,y)$.
   c. Illustrate this point's position on a plot of the unit circle.

3. Consider the point $\left(\frac{5}{13}, \frac{12}{13}\right)$
   a. Verify that the point lies on the unit circle.
   b. Use the diagram at right to approximate to the nearest tenth a value of t so that $\cos(t) = \frac{5}{13} \approx 0.38$
   c. Approximate to the nearest tenth the interval in the first quadrant where $\frac{5}{12} \le \tan(t) \le \frac{12}{5}$



4. Recall that a function is even if $f(-x) = f(x)$ and odd if $f(-x) = -f(x)$. Of the six trigonometric functions: sin(x), cos(x), tan(x), sec(x), csc(x) and cot(x)
   a. Which functions are even?
   b. Which functions are odd?

5. Consider the point $\left(\frac{8}{17}, \frac{15}{17}\right)$
   a. Verify that the point lies on the unit circle.
   b. Use the diagram at right to approximate to the nearest tenth a value of t so that $\cos(t) = \frac{8}{17} \approx 0.47$
   c. Approximate to the nearest tenth a value of t so that $\tan(t) = \frac{8}{15}$



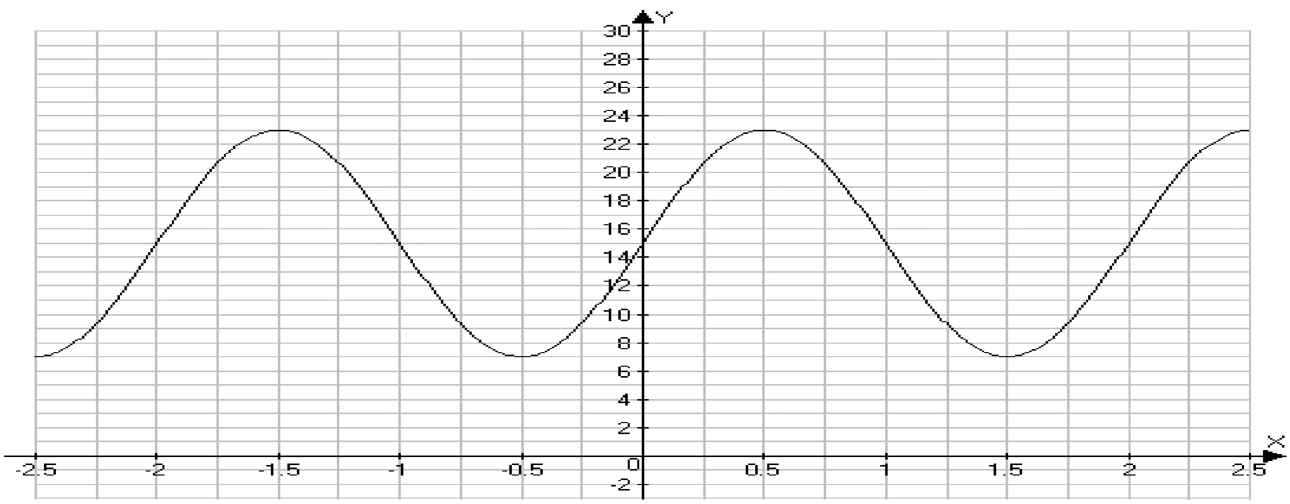
6. Suppose that $\cos(t) = \frac{\sqrt{91}}{100}$ and point and sin(t) < 0. Find $\sin(t), \tan(t), \sec(t), \csc(t)$ and $\cot(t)$.

7. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for *t* is in quadrant III.

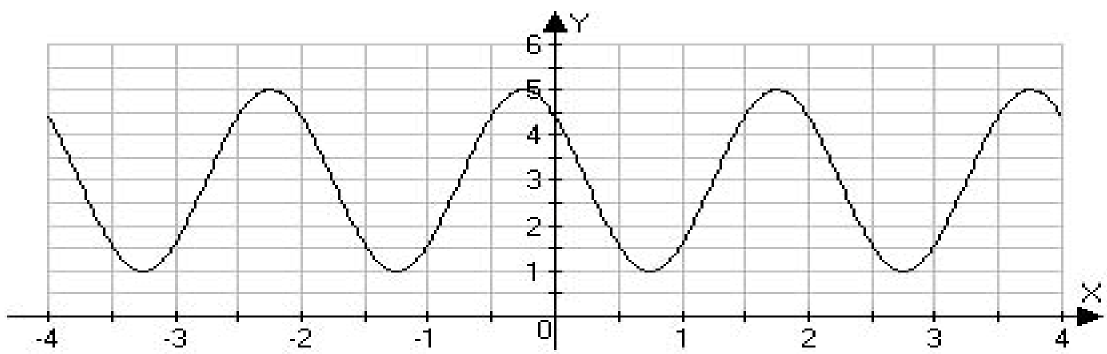
8. Find the amplitude, period and phase shift of $y = 5 + 5\sin\left(20\pi\left(x - \frac{1}{50}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

9. Find an equation for the sinusoid whose graph is shown:

a.



b.



10. Consider the function $f(x) = \tan\left(\frac{\pi}{2}x\right)$.

   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates where $y = 0$ and where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

11. Consider the function $f(x) = \tan\left(\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right)$.

   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates where $y = 0$ and where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

12. Suppose $\cos t = 9/28$ and *t* is in the first quadrant. Find the following:

   a. $\cos(t + \pi)$ b. $\cos\left(t + \frac{\pi}{2}\right)$ c. $\cos\left(\frac{\pi}{2} - t\right)$

13. Suppose *sin t* = 16/65 and *t* is in the first quadrant. Find the following:

a. $\sin(t+\pi)$ b. $\sin\left(t+\frac{\pi}{2}\right)$ c. $\sin\left(\frac{\pi}{2}-t\right)$

14. Complete the table of values for $f(t)=\cos(\pi t)+2\sin(\pi t)$, plot the points and sketch a graph.

t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$\cos(\pi t)$									
$2\sin(\pi t)$									
$f(t)$									

15. The Millennium Wheel rotates once every 30 minutes. Its highest point is about 135 meters above the ground and the lowest point is about 5 meters above the ground. Write a function that gives the height of a rider *t* minutes after boarding the Millennium Wheel.

Chapter 3 – Fair Game Problem Solutions

1. For arclength $t=\frac{29\pi}{6}$ traced counterclockwise along the unit circle from (1,0)

   a. Find the reference number for *t*.

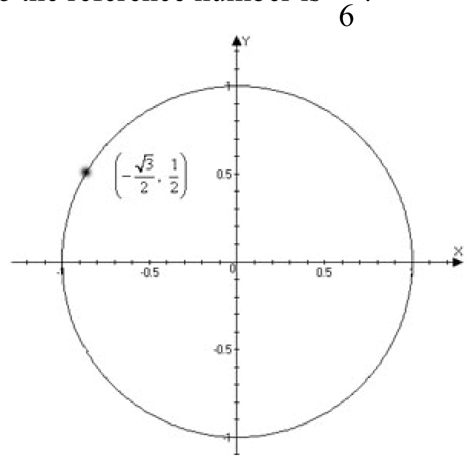
   SOLN: $t=\frac{29\pi}{6}=\frac{(12+12+5)\pi}{6}=2\pi+2\pi+\frac{5\pi}{6}$ so the reference number is $\frac{\pi}{6}$.

   b. What are the coordinates of the terminal point *P*(*x*,*y*)?

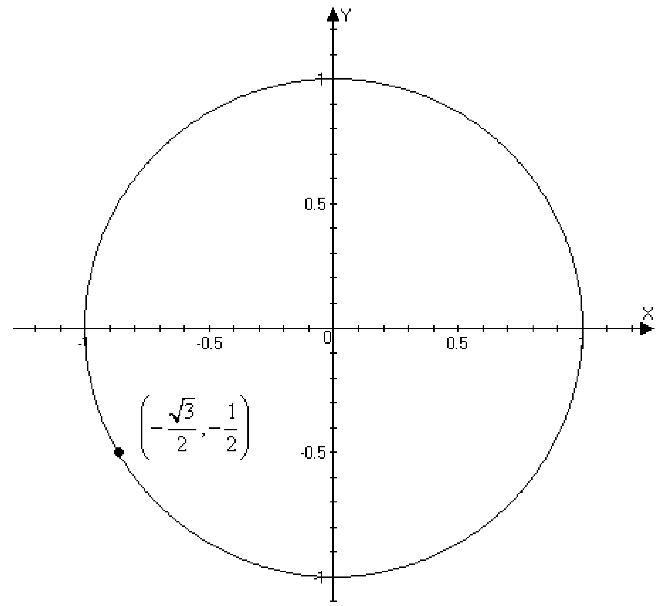
   SOLN: Since this point is in the 2nd quadrant, *x* is negative, but $y>0$. $x=-\cos\left(\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}$ and $y=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$.

   c. Illustrate this point's position on a plot of the unit circle.

   ANS: The point $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ is shown



2. For arclength $t = \frac{31\pi}{6}$ extending counterclockwise along the unit circle from (1,0)

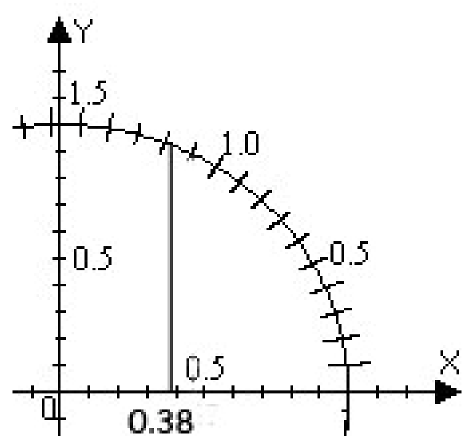


d. Find the reference number for *t*.
ANS:
$$t = \frac{31\pi}{6} = \frac{(12+12+6+1)\pi}{6} = 2\pi + 2\pi + \pi + \frac{\pi}{6}$$
so the reference number is $\frac{\pi}{6}$.

e. Find the coordinates of the terminal point *P*(*x*,*y*).
ANS: Since this point is in the third quadrant, both *x* and *y* are negative and so
$x = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ and
$y = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$.

f. Illustrate this point's position on a plot of the unit circle.
ANS: The point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

3. Consider the point $\left(\frac{5}{13}, \frac{12}{13}\right)$



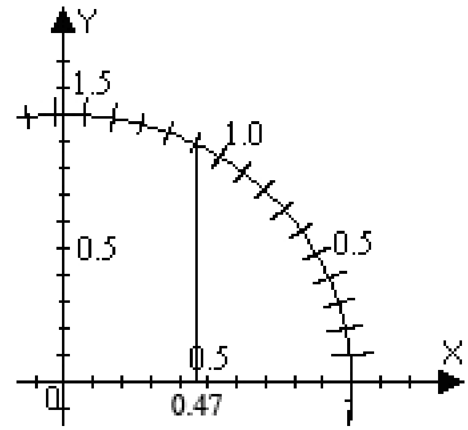
a. Verify that the point lies on the unit circle.
ANS: $\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{25}{169} + \frac{144}{169} = \frac{169}{169} = 1$

b. Use the diagram at right to approximate to the nearest tenth a value of *t* so that
$\cos(t) = \frac{5}{13} \approx 0.38$
ANS: A vertical segment is drawn from 0.38 on the *x*-axis intersects the circle at *t* between 1.1 and 1.2: closer to *t* = 1.2. Indeed, arccos(5/13) is approximately 1.176

c. Approximate to the nearest tenth the interval in the first quadrant where $\frac{5}{12} \le \tan(t) \le \frac{12}{5}$
ANS: If cos(*t*) = 5/13 and sin(*t*) = 12/13, then tan(*t*) = 12/5. Since cot(*t*) = cos(*t*)/sin(*t*) = 5/12 and tan(π/2 – *t*) = cot(*t*). So choose *t* = 1.6 – 1.2 = 0.4 so that the approximate *t* interval we seek is *t* between 0.4 and 1.2.

4. Recall that a function is even if $f(-x) = f(x)$ and a function is odd if $f(-x) = -f(x)$. Of the six trigonometric functions, which are even and which are odd?
SOLN: Only cos(*t*) and sec(*t*) are even . The other four are odd.

5. Consider the point $\left(\frac{8}{17},\frac{15}{17}\right)$



   d. Verify that the point lies on the unit circle.

   ANS: $\left(\frac{8}{17}\right)^2+\left(\frac{15}{17}\right)^2=\frac{64}{289}+\frac{225}{289}=\frac{289}{289}=1$

   e. Use the diagram at right to approximate to the nearest tenth a value of t so that $\cos(t)=\frac{8}{17}\approx 0.47$

   ANS: A vertical segment is drawn from 0.47 on the *x*-axis intersects the circle at *t* near 1.1

   f. Approximate to the nearest tenth a value of *t* so that $\tan(t)=\frac{8}{15}$

   ANS: Since cot(*t*) = cos(*t*)/sin(*t*) = 8/15 and tan(π/2 – *t*) = cot(*t*). So choose *t* = 1.6 – 1.1 = 0.5

6.

7. Suppose that $\cos(t)=\frac{\sqrt{91}}{100}$ and point and sin(*t*) < 0.

   Find $\sin(t),\tan(t),\sec(t),\csc(t)$ and $\cot(t)$.

   ANS: $\sin(t)=-\sqrt{1-\cos^2 t}=-\sqrt{1-\left(\frac{\sqrt{91}}{100}\right)^2}=-\sqrt{1-\frac{91}{10000}}=-\sqrt{\frac{10000-91}{10000}}=-\sqrt{\frac{9909}{10000}}=-\frac{3\sqrt{1101}}{100}$

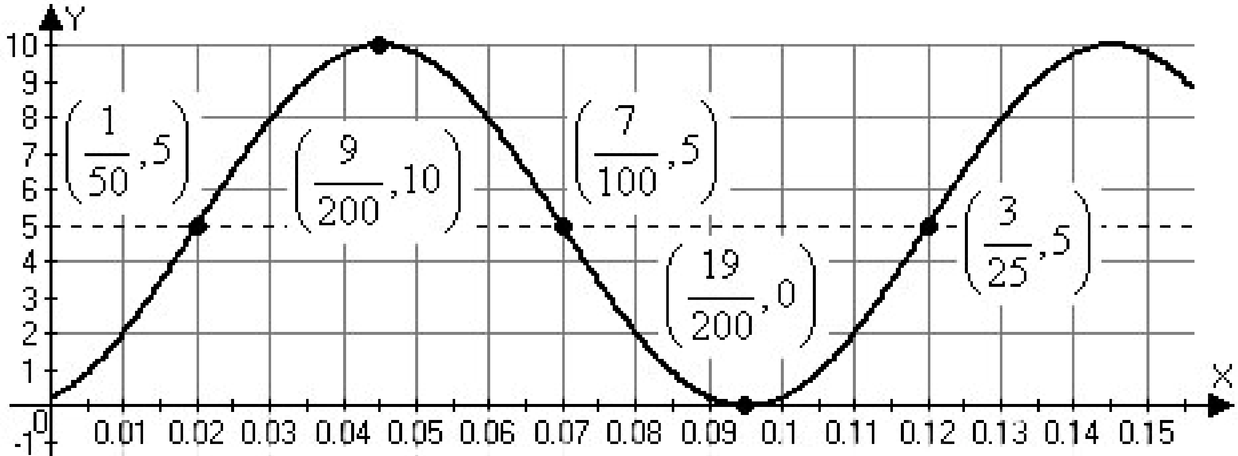
   $\tan(t)=-\frac{3\sqrt{1101}}{\sqrt{91}}=-\frac{3\sqrt{100191}}{91}$; $\sec(t)=\frac{100\sqrt{91}}{91}$; $\csc(t)=-\frac{100\sqrt{1101}}{3303}$; $\cot(t)=-\frac{\sqrt{100191}}{3303}$

8. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for *t* is in quadrant III.

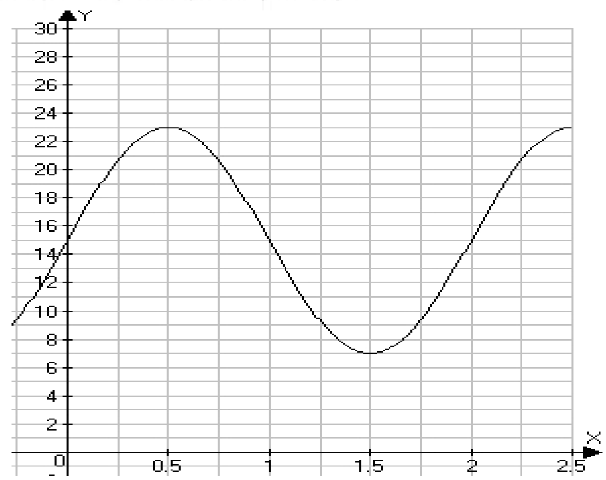
   ANS: Starting with $\cos^2 t+\sin^2 t=1$, divide through by $\cos^2 t$ to obtain $1+\tan^2 t=\sec^2 t$. Since sec(*t*) is negative in quadrant III, $\sec t=-\sqrt{1+\tan^2 t}$

a. Find the amplitude, period and phase shift of $y=5+5\sin\left(20\pi\left(x-\frac{1}{50}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

   ANS: The amplitude is 5, the period is 1/10 and the phase angle is 1/50.
   Graph is shown below. The starting point and endpoint are $\left(\frac{1}{50},5\right)$ and $\left(\frac{3}{25},5\right)$. The halfway point is $\left(\frac{7}{100},5\right)$ while the peak and trough are, respectively, $\left(\frac{9}{200},10\right)$ and $\left(\frac{19}{200},0\right)$.



9. Find an equation for the sinusoid whose graph is shown:



   a. ANS: The lowest point is at y=7 and the highest point is at 23 so the line of equilibrium is at the average of these: $y = (7+23)/2 = 15$. and the amplitude is $(23 - 7)/2 = 8$.

      The two peaks shown in the graph here are where $x = 0.5$ and $x = 2.5$, so the period is $2.5 - 0.5 = 2$. Thus an equation for the sinusoid is $y = 15 + 8\sin(\pi x)$.

   b. $y = 3 + \sin\left(\pi\left(x + \frac{3}{4}\right)\right)$

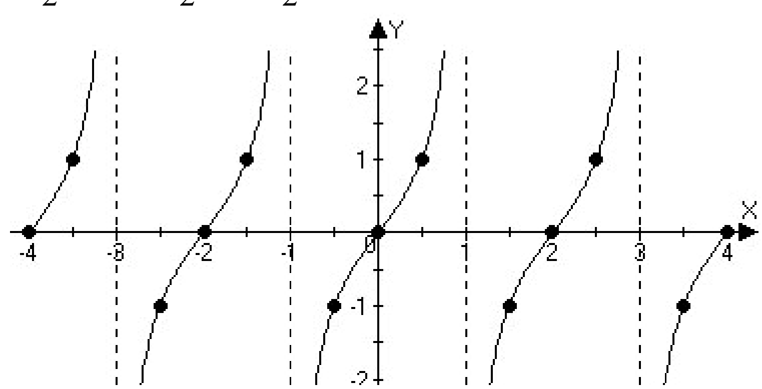
10. Consider the function $f(x) = \tan\left(\frac{\pi}{2}x\right)$.

   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.

      ANS: We want the input to the tangent to be $\pm\frac{\pi}{2}$, that is $\frac{\pi}{2}x = \pm\frac{\pi}{2} \Leftrightarrow x = \pm 1 \Leftrightarrow \boxed{x = -1 \text{ or } 1}$

   b. Find x-coords where $y = 0$ and $y = \pm 1$.

      SOLN: If x is any even integer then $f(2k) = \tan(k\pi) = 0$, which is true for any integer value for k.



      Also We want to find where the input to the tangent function is equal to $\pm\frac{\pi}{4}$, that is $\frac{\pi}{2}x = \pm\frac{\pi}{4} \Leftrightarrow x = \pm\frac{1}{2}$

   c. The pattern of even integer $x = 2k$ intercepts and points: $(2k-1/2, -1)$ and $(2k+1/2, 1)$ is evident in the graph. Also note that $x = 2k + 1$ are vertical asymptotes for $k \in \mathbb{Z}$

11. Consider the function $f(x) = \tan\left(\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right)$.

   c. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.

ANS: We want the input to the tangent to be $\pm\frac{\pi}{2}$, that is

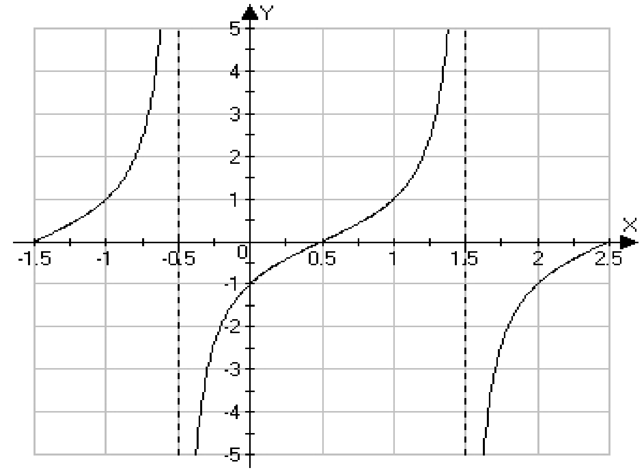
$$\frac{\pi}{2}\left(x-\frac{1}{2}\right)=\pm\frac{\pi}{2} \Leftrightarrow x-\frac{1}{2}=\pm 1 \Leftrightarrow \boxed{x=\frac{1}{2}\pm 1=-\frac{1}{2} \text{ or } \frac{3}{2}}$$

d. Find x-coords where $y = 0$ and $y = \pm 1$.
ANS: We want to find where the input to the tangent function is equal to $\pm\frac{\pi}{4}$, that is

$$\frac{\pi}{2}\left(x-\frac{1}{2}\right)=\pm\frac{\pi}{4} \Leftrightarrow x-\frac{1}{2}=\pm\frac{1}{2}.$$

$$\Leftrightarrow \boxed{x=0 \text{ or } x=1}$$

e. Graph of the function showing how it passes through the points where y = -1, y = 0, y = 1 and how it approaches the vertical asymptotes.



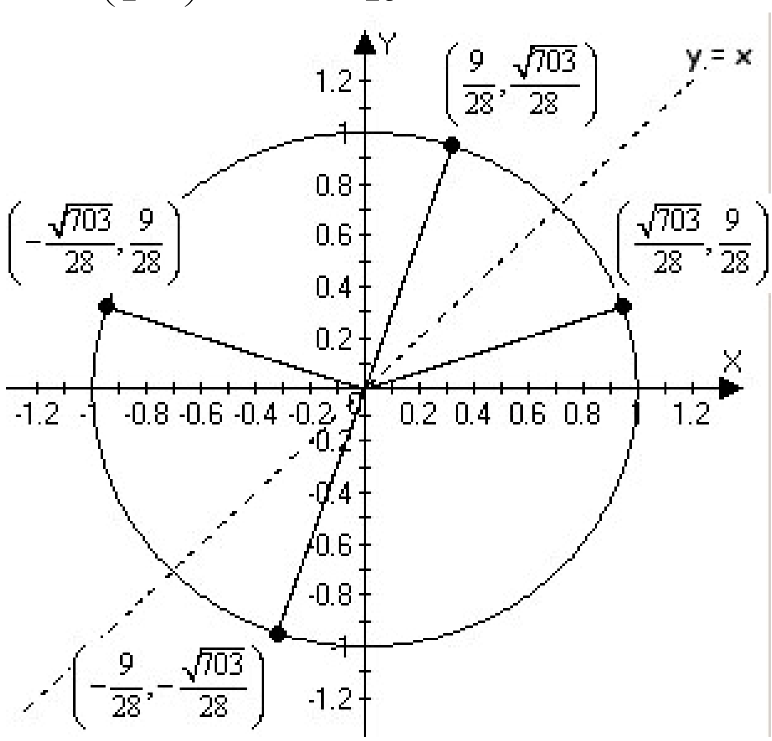
12. Suppose *sin t* = 9/28 and *t* is in the first quadrant. Find the following:

a. $\cos(t+\pi)=-\frac{9}{28}$

b. $\cos\left(t+\frac{\pi}{2}\right)=\sin(t)=\sqrt{1-\left(\frac{9}{28}\right)^2}=\sqrt{1-\frac{81}{784}}=\sqrt{\frac{703.}{784}}=\frac{\sqrt{703.}}{28}$

Note that 703 = 19*37 is not prime but is square free.

c. $\cos\left(\frac{\pi}{2}-t\right)=\sin(t)=\frac{\sqrt{703}}{28}$

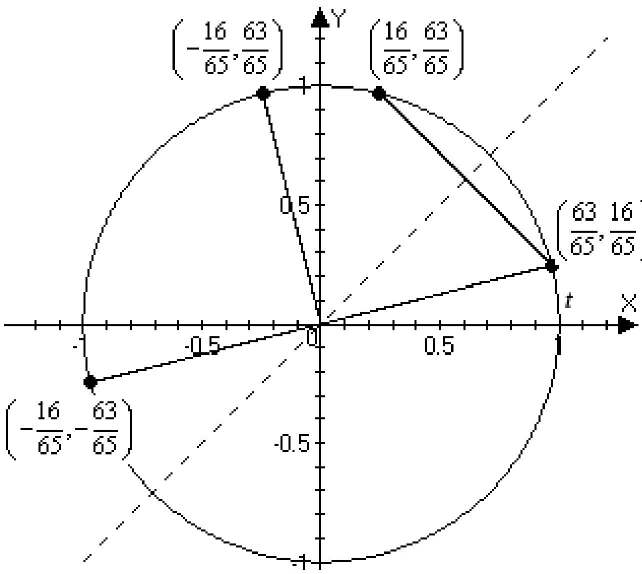


13. Suppose *sin t* = 16/65 and *t* is in the first quadrant. Find the following:

d. $\sin(t+\pi) = -\frac{16}{65}$

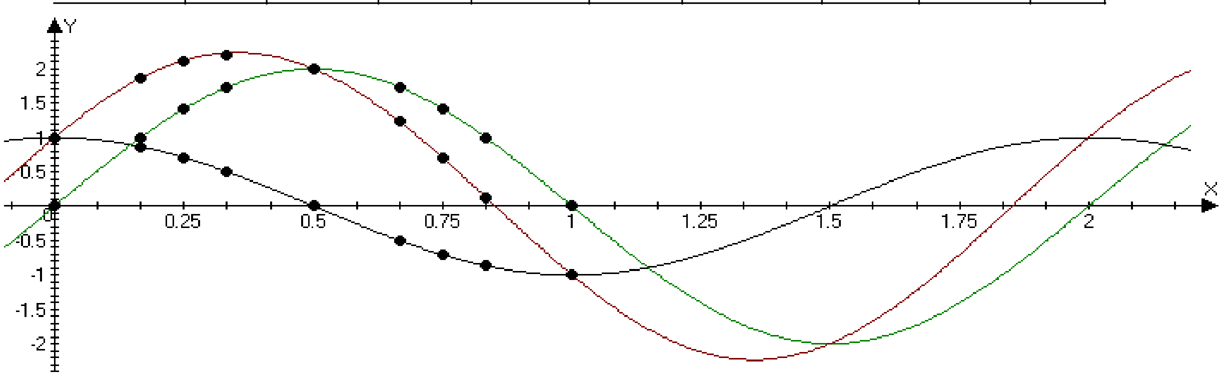
e. $\sin\left(t+\frac{\pi}{2}\right) = -\cos(t) = -\sqrt{1-\left(\frac{16}{65}\right)^2} = -\sqrt{1-\frac{256}{4225}} = -\sqrt{\frac{3969}{4225}} = \frac{63}{65}$

f. $\sin\left(\frac{\pi}{2}-t\right) = \frac{63}{65}$



14. Complete the table of values for $f(t) = \cos(\pi t) + 2\sin(\pi t)$, plot the points and sketch a graph.

t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$\cos(\pi t)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$2\sin(\pi t)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0
$f(t)$	1	$1+\frac{\sqrt{3}}{2}$	$\frac{3\sqrt{2}}{2}$	$\frac{1}{2}+\sqrt{3}$	2	$-\frac{1}{2}+\sqrt{3}$	$\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{3}}{2}$	-1



15. The Millennium Wheel rotates once every 30 minutes. Its highest point is about 135 meters above the ground and the lowest point is about 5 meters above the ground. Write a function that gives

the height of a rider t minutes after boarding the Millennium Wheel.
ANS: $h(t) = 70 - 65\cos(\pi t / 15)$