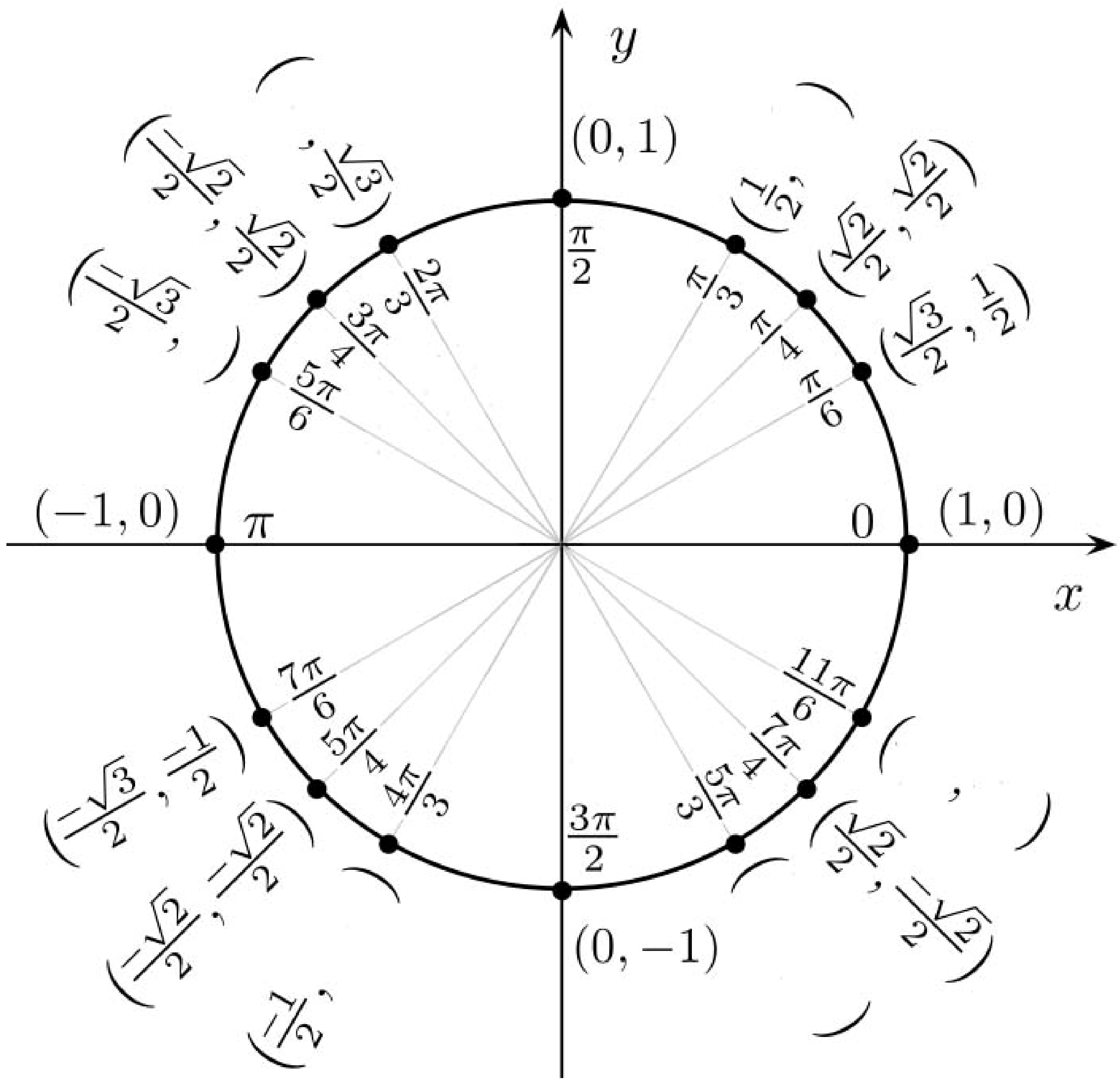
Math 5 – Trigonometry – Chapter 3 Test – spring '11 Name__________________________

Show work for credit. Unless otherwise directed, write responses on separate paper. Don't use a calculator.

1. There are 8 blanks in the coordinates of the unit circle points illustrated below. Fill them in.



2. Consider the terminal point on the unit circle determined by traversing the circumference a distance $t = \dfrac{38\pi}{3}$ counterclockwise from the point (1,0).

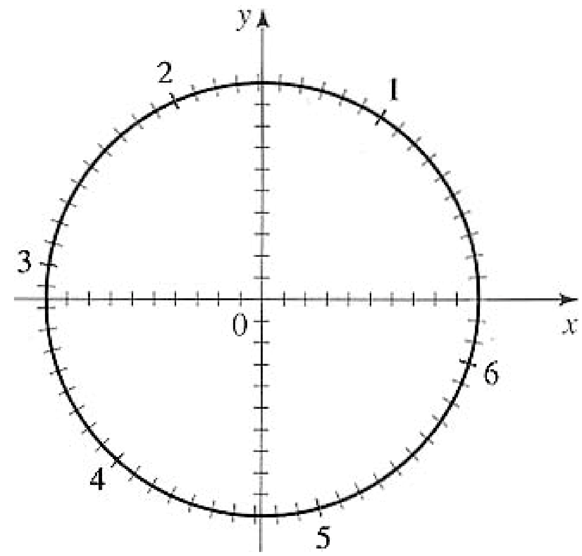
a. How many complete rotations around the circle is this?

b. Draw the unit circle and locate the terminal point.

c. What are the (*x*,*y*)coordinates of the terminal point?

3. Suppose the *y*-coordinate of a terminal point *P*(*t*) on the unit circle is $y = \frac{4\sqrt{6}}{11}$ and the terminal point is in the second quadrant. Find sin(*t*), cos(*t*), and tan(*t*).

4. Use the diagram at right showing the number line wrapped around the circumference of the unit circle to locate the point (highlight it on the diagram here) and approximate, to the nearest tenth,
   a. $\cos(3.5)$
   b. $\sin(3.5)$
   c. $\tan(3.5)$



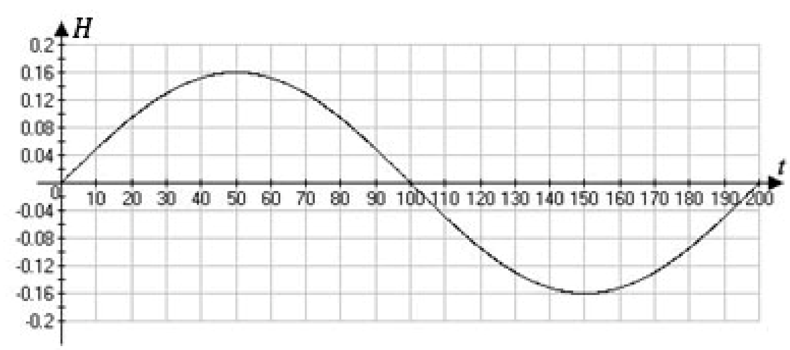
5. What kind of number is $\tan(6.3)$? Why?
   a. Undefined
   b. Negative of large magnitude
   c. Positive of large magnitude
   d. Negative of small magnitude
   e. Positive of small magnitude

6. Consider the sinusoid $y = 3 + 4\cos(2\pi(x-1))$
   a. Find the line of equilibrium, amplitude, period, and phase shift of the function.
   b. Give a table of values for five points evenly spaced along the domain of the function.
   c. Sketch a graph clearly showing coordinates of at least 5 points of one oscillation.

7. Consider the function $f(x) = \tan\left(2\left(x - \frac{\pi}{6}\right)\right)$.
   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates where *y* = 0 and where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

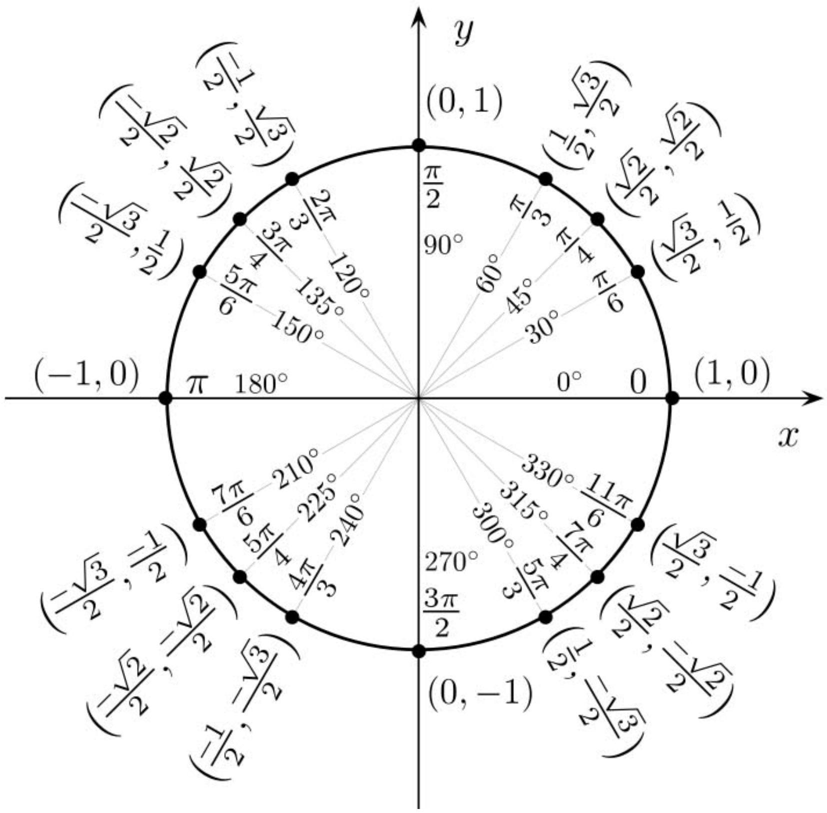
8. Suppose *sin t* = 4/5 and *t* is in the first quadrant. Find the following:
   a. $\sin(\pi - t)$ b. $\cos\left(t + \frac{\pi}{2}\right)$ c. $\sin\left(\frac{\pi}{2} - t\right)$

9. The graph to the right shows the height, *H*, (in meters) above sea level of a bird floating in the waves, as a function of time (in seconds).
   a. Find the amplitude and period of the function.
   b. Give a formula for the function, as a sinusoid in *t*.
   c. What is the frequency of oscillation, in cycles per second?



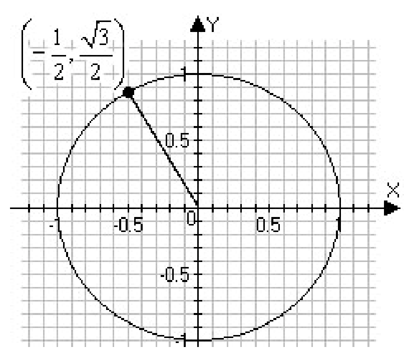
Math 5 – Trigonometry – Chapter 4 Test Solutions – spring ’11

1. There are 8 blanks in the coordinates of the unit circle points illustrated below. Fill them in.



2. Consider the terminal point on the unit circle determined by traversing the circumference a distance $t = \frac{38\pi}{3}$ counterclockwise from the point (1,0).

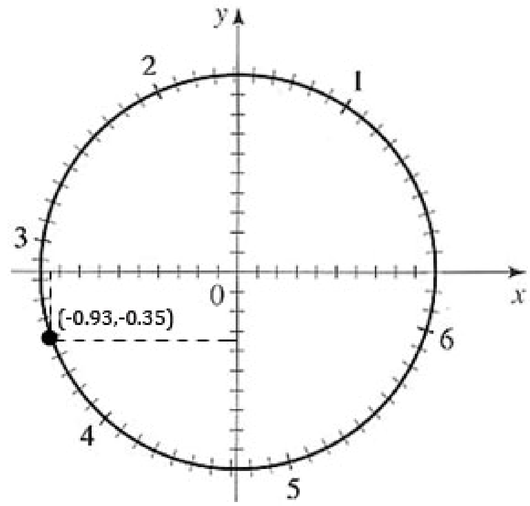
   a. SOLN: This is 6 rotations $t = \frac{38\pi}{3} = \frac{36\pi + 2\pi}{3} = 12\pi + \frac{2\pi}{3}$ is coterminal with $\frac{2\pi}{3}$ where the coordinates are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.



3. Suppose the y-coordinate of a terminal point $P(t)$ on the unit circle is $y = \frac{4\sqrt{6}}{11}$ and the terminal point is in the second quadrant. Find sin(t), cos(t), and tan(t).

   SOLN: $x^2 + y^2 = 1$ on the unit circle, so $x^2 + \left(\frac{4\sqrt{6}}{11}\right)^2 = x^2 + \frac{96}{121} = 1 \Leftrightarrow x^2 = \frac{25}{121}$ and since x is in the second quadrant, $x = -5/11$. Thus $\sin(t) = \frac{4\sqrt{6}}{11}$, $\cos(t) = -\frac{5}{11}$ and $\tan(t) = -\frac{4\sqrt{6}}{5}$.

4. Use the diagram at right showing the number line wrapped around the circumference of the unit circle to locate the point (highlight it on the diagram here) and approximate, to the nearest tenth,
   a. $\cos(3.5) \approx -0.9$
   b. $\sin(3.5) \approx -0.35$
   c. $\tan(3.5) \approx 0.4$



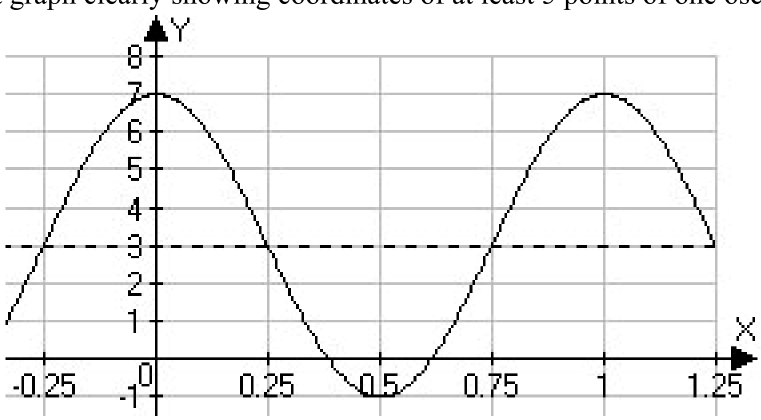
5. What kind of number is $\tan(6.3)$? Why?
   SOLN: Since 6.3 is slightly larger than 2π, the reference number is very small and positive, so sin(6.3) is very small and positive while cos(6.3) is just less than 1, so tan(6.3) is positive of small magnitude.

6. Consider the sinusoid $y = 3 + 4\cos(2\pi(x-1))$
   a. Find the line of equilibrium, amplitude, period, and phase shift of the function.
   SOLN: $y = 3 + 4\cos(2\pi(x-1))$ has line of equilibrium $y = 3$, amplitude 4, period = 1 and phase shift = 0. You may have the phase shift = 1, but it doesn't make sense to have a whole wavelength phase shift.
   b. Give a table of values for five points evenly spaced along the domain of the function.
   SOLN:

x	0	¼	½	¾	1
y	7	3	–1	3	7

   c. Sketch a graph clearly showing coordinates of at least 5 points of one oscillation.

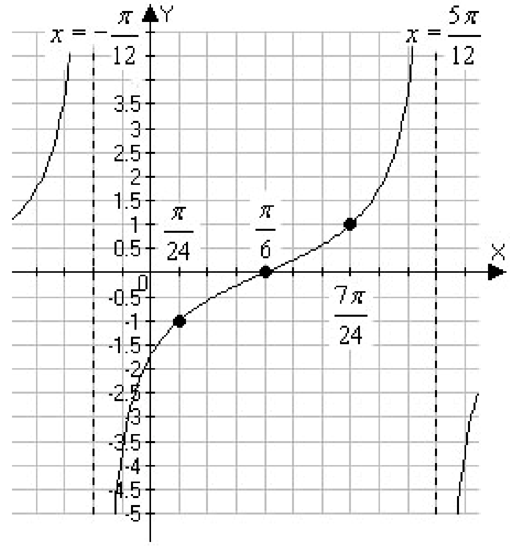
SOLN:



7. Consider the function $f(x) = \tan\left(2\left(x - \frac{\pi}{6}\right)\right)$.

   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   SOLN;
   $$-\frac{\pi}{2} < 2\left(x - \frac{\pi}{6}\right) < \frac{\pi}{2} \Leftrightarrow -\frac{\pi}{4} < x - \frac{\pi}{6} < \frac{\pi}{4}$$
   $$\Leftrightarrow -\frac{\pi}{12} < x < \frac{5\pi}{12}$$

   b. Find the *x*-coordinates where $y = 0$ and $y = \pm 1$.
   SOLN: $$2\left(x - \frac{\pi}{6}\right) = \pm\frac{\pi}{4} \Leftrightarrow x - \frac{\pi}{6} = \pm\frac{\pi}{8}$$
   $$\Leftrightarrow x = \frac{\pi}{6} \pm \frac{\pi}{8} = \frac{\pi}{24} \text{ or } \frac{7\pi}{24}$$



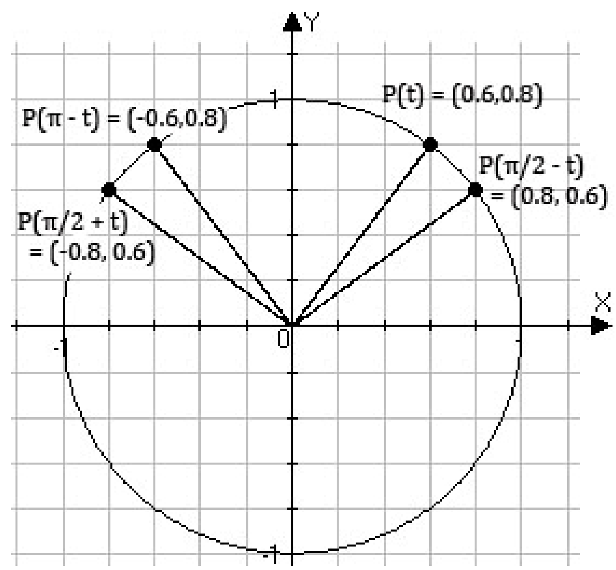
c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes. (SOLN shown above)

8. Suppose $\sin t = 4/5$ and t is in the first quadrant. Find the following:

   a. $\sin(\pi - t)$ SOLN: $\sin(\pi - t) = \frac{4}{5}$

   b. $\cos\left(t + \frac{\pi}{2}\right)$ SOLN: $\cos\left(t + \frac{\pi}{2}\right) = -\frac{4}{5}$

   c. $\sin\left(\frac{\pi}{2} - t\right)$ SOLN: $\sin\left(\frac{\pi}{2} - t\right) = \frac{3}{5}$



9. The graph to the right shows the height, *H*, (in meters) above sea level of a bird floating in the waves, as a function of time (in seconds).

   a. Find the amplitude and period of the function.
   SOLN: The amplitude is 0.16 and the period = 200

   b. Give a formula for the function, as a sinusoid in *t*.
   SOLN: $y = 0.16\sin\left(\frac{\pi t}{100}\right)$

   c. What is the frequency of oscillation, in cycles per second?
   SOLN: One oscillation per 200 seconds, or 0.005 Hz