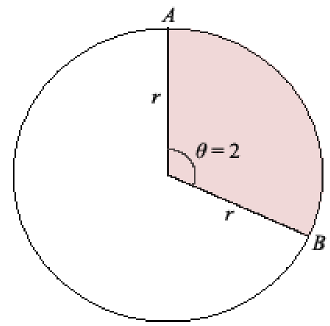
Math 5 – Trigonometry – Chapter 4 Test – spring '11 Name___________________________

Show your work for credit. Write all responses on separate paper.

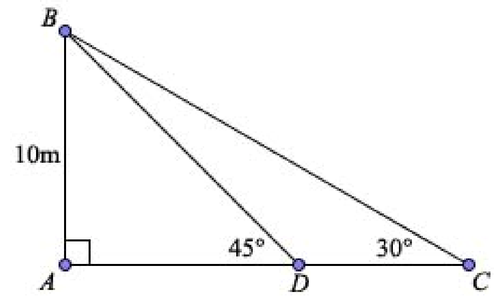
1. In the circle at right, arc length AB subtends a central angle of 2 radians.



   a. Suppose arc AB is 300 meters. What is the radius?

   b. Suppose the area of sector AOB is 5 square millimeters. What is the radius?

2. Given that AB = 10 meters in the diagram at right, find the length of CD.



3. A 7 inch diameter disc rotates at 45 revolutions per minute.

   a. Find the angular speed of the disc, in degrees per second.

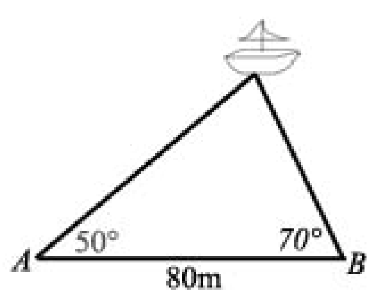
   b. What is the linear speed of the rim of the disc, in feet per second?

4. Phoebe is the outermost moon of Saturn and revolves at a distance of 23.5×10^{6} km from Saturn once every 550 days. Find the linear speed of Phoebe relative to Saturn in meters per second.

5. Suppose $f(x) = x^2 + 1$ and $g(x) = \tan^{-1}(x)$.
   a. Evaluate $f\left(g(1)\right)$
   b. Approximate to the nearest hundredth $g\left(f(1)\right)$

6. Draw a triangle with an angle of 1 radian at vertex A and a side of length b = 10 adjacent to A.
   a. What is the minimum length of side a (opposite A) that will make a triangle?
   b. What range of values of a will allow for two different triangles to be constructed?

7. Observers of a ship from the shoreline AB are located at A and B (see diagram at right) and measure the ship to shore angles of the ship from point 80 meters apart. How far from shore is the ship?



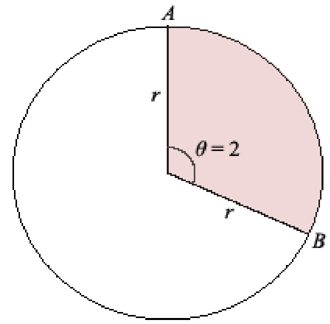
8. In ΔABC, $a = 4$, $b = 14$ and $c = 11$.
   a. Approximate to the nearest hundredth the radian measures of the interior angles of the triangle.
   b. Find the area of the triangle in simplest radical form.

Math 5 – Trigonometry – Chapter 4 Test Solutions – spring ’11

1. In the circle at right, arc length AB subtends a central angle of 2 rads.
   a. Suppose AB is 300 meters. What is the radius?

   SOLN: Using the definition of radian measure, $\theta = \frac{s}{r}$ we have

   $2 = \frac{300}{r} \Leftrightarrow r = 150$ meters.



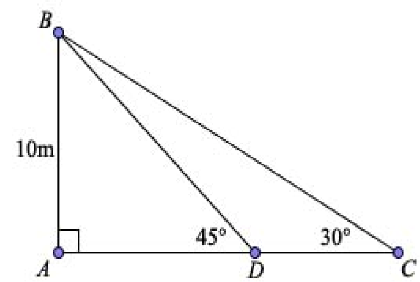
   b. If the area of sector AOB is 5 square millimeters then, since the area of the sector is proportional to the central angle,

   $\frac{\theta}{2\pi} = \frac{A}{\pi r^2} \Leftrightarrow \frac{2}{2\pi} = \frac{5}{\pi r^2} \Leftrightarrow r^2 = 5$, So $r = \sqrt{5}$ mm

   is $\frac{5\pi}{3}\frac{30^2}{2} + 2\left(\frac{2\pi}{3}\frac{10^2}{2}\right) = 750\pi + \frac{200\pi}{3} = \frac{2450\pi}{3} \approx 2565.6 \text{ ft}^2$.

2. Given that AB = 10 meters in the diagram at right, find the length of CD

   SOLN: $CD = AC - AD = 10\sqrt{3} - 10 = 10\left(\sqrt{3} - 1\right) \approx 7.32$ meters



3. A disc 7 inches in diameter rotates at a rate of 45 revolutions per minute.
   a. What is the angular speed of the disc, in degrees per second?

   SOLN: $\frac{45 \text{ rotations}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rotation}} = \frac{90\pi}{\text{min}} \times \frac{180^\circ}{\pi} \times \frac{1 \text{ min}}{60 \text{sec}} = \frac{270^\circ}{\text{sec}}$

   b. What is the linear speed of the rim of the disc, in feet per second?

   SOLN: Converting the angular speed to radians/sec, $\omega = \frac{90\pi}{\text{min}} \times \frac{1 \text{min}}{60 \text{sec}} = \frac{3\pi}{2 \text{sec}}$ and using $v = \omega r$, we have $v = \frac{3\pi}{2 \text{sec}}\left(\frac{7}{2}\text{inch}\right) = \frac{21\pi \text{ inches}}{4 \text{sec}} \times \frac{1 \text{ foot}}{12 \text{ inches}} = \frac{7\pi}{16} \text{fps} \approx 1.374 \text{ fps}$

4. Phoebe is the outermost moon of Saturn and revolves at a distance of 23.5×10^6 km from Saturn once every 550 days. Find the linear speed of Phoebe relative to Saturn in meters per second

   $v = \omega r = \frac{1 \text{rot}}{550 \text{days}} \times \frac{2\pi \text{ rad}}{\text{rot}} \times 23.5 \times 10^6 \text{km} = \frac{23.5\pi \times 10^6 \text{km}}{275 \text{days}} \approx 267000 \frac{\text{km}}{\text{day}} \times \frac{1 \text{day}}{24 \text{hr}} \times \frac{1 \text{hr}}{3600 \text{sec}} \approx 3100 \text{m/s}$

5. Suppose $f(x) = x^2 + 1$ and $g(x) = \tan^{-1}(x)$.
   a. Evaluate $f\left(g(1)\right)$

   SOLN: $f\left(g(1)\right) = f\left(\arctan(1)\right) = f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + 1$

   b. Approximate to the nearest hundredth $g\left(f(1)\right)$

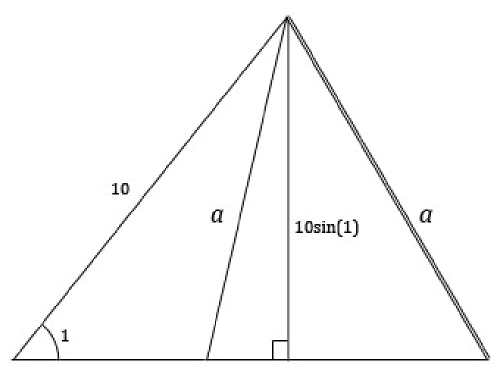
   SOLN: $g\left(f(1)\right) = g\left(1^2 + 1\right) = g(2) = \arctan(2) \approx 1.11 \approx 63.43^\circ$

6. Draw a triangle with an angle of 1 radian at vertex A and a side of length b = 10 adjacent to A.
   a. What is the minimum length of side a (opposite A) that will make a triangle?

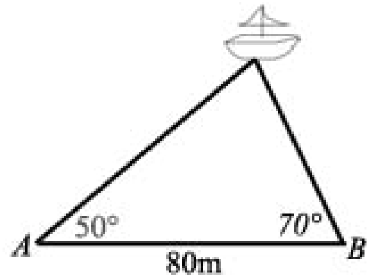
   SOLN: As shown in the diagram, the minimum is 10sin(1).

   b. What range of values of a will allow for two different triangles to be constructed?

   SOLN: a must be less than 10, so $10\sin(1) < a < 10$.



7. Observers of a ship from the shoreline *AB* are located at *A* and *B* (see diagram at right) and measure the ship to shore angles of the ship from point 80 meters apart. How far from shore is the ship?



SOLN: By the law of sines, $\frac{b}{\sin 70^\circ} = \frac{80}{\sin 60^\circ} \Leftrightarrow b = \frac{160\sin 70^\circ}{\sqrt{3}}$

so that the ship to shore distance is $\frac{160\sin 70^\circ}{\sqrt{3}}\sin 50^\circ \approx 66.5\,\text{m}$

8. In ΔABC, $a = 4$, $b = 14$ and $c = 11$.
   a. Approximate to the nearest hundredth the radian measures of the interior angles of the triangle.
   SOLN: You can get any of the angles first using the law of cosines:
   $$\cos C = \frac{4^2+14^2-11^2}{2(4)(14)} = \frac{13}{16} \Leftarrow C = \cos^{-1}\left(\frac{13}{16}\right) \approx 35.66^\circ \approx 0.62 \text{ or}$$
   $$\cos B = \frac{4^2+11^2-14^2}{2(4)(11)} = \frac{-59}{88} \Leftarrow B = \cos^{-1}\left(\frac{-59}{88}\right) \approx 132.10^\circ \approx 2.31 \text{ or}$$
   $$\cos A = \frac{14^2+11^2-4^2}{2(14)(11)} = \frac{301}{308} \Leftarrow A = \cos^{-1}\left(\frac{301}{308}\right) \approx 12.24^\circ \approx 0.21$$ - These 3 angles sum to 3.14
   Then you can use the law of sines to find a second angle and the theorem for the sum of the interior angles of any triangles will most easily yield the third.
   b. Find the area of the triangle in simplest radical form.
   SOLN: The semiperimeter is (11+15+10)/2 = 18 so Heron's formula yields
   $$\sqrt{\frac{29}{2}\left(\frac{21}{2}\right)\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)} = \frac{7\sqrt{87}}{4}$$