Math 5 – Trigonometry – Spring '11 – Chapter 2 Test Name____________________________

Instructions: Show all work for credit. Write all responses on separate paper. Do not use a calculator.

1. Compute and simplify the average rate of change of $f(x)=\dfrac{1}{x-2}$ over the interval [0, *h*]. *Hint: Recall that the average rate of change over [a,b] is the slope of the line from [a, f(a)] to [b, f(b)].*

2. Consider the quadratic function $f(x)=3x^2-6x+2$
   a. Express the quadratic function in standard (vertex) form: $y=a(x-h)^2+k$
   b. Find the coordinates of the *x*-intercepts.
   c. Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.

3. Consider the piecewise defined function $f(x)=\begin{cases} x & x<1 \\ 1+2\sqrt{x-1} & \text{if } x\ge 1 \end{cases}$
   a. Make a table of values and sketch a graph for the function.
   b. Show that *f* is one-to-one and find a formula for the inverse function.

4. Consider the quadratic $f(x)=-2x^2+4x+3$
   a. Express the quadratic function in standard form.
   b. Sketch its graph showing the position of the vertex.
   c. What sequence of

      (i) vertical shift,

      (ii) reflection,

      (iii) vertical shrink, and

      (iv) horizontal shift

   would be required to transform this function to $y=x^2$?

5. Given the graph of $y = f(x)$ shown at right and the given transformation, tabulate the transformed coordinate values of the key points in the graph. Then plot the given transformation on the paper at right.

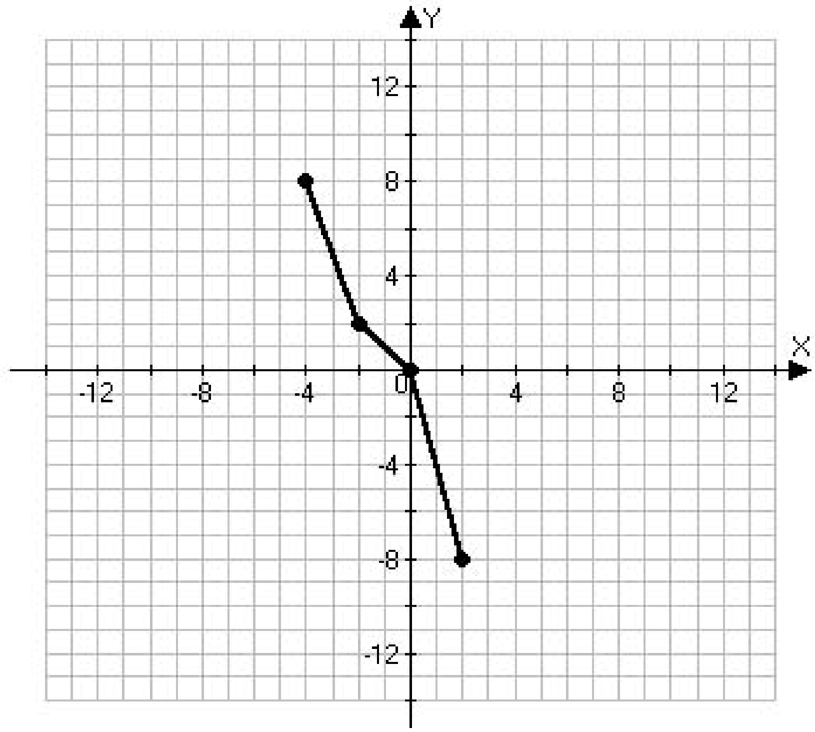
x	-4	-2	0	2
$f(x)$	8	2	0	-8

a. $y = f(x-4)+3$

x				
$f(x-4)+3$				

b. $y = \frac{3}{2} f(x+8)$

x				
$\frac{3}{2} f(x+8)$				



6. The surface area of a sphere is a function of the radius according to $S = f(r) = 4\pi r^2$ and the volume of a sphere is a function of the radius according to $V = g(r) = \frac{4}{3}\pi r^3$.
Find a function that gives the surface area, *S*, as a function of the Volume, *V*.

7. Suppose $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x}$.
   a. What is the domain of f?
   b. What is the range of f?
   c. What is the domain of g?
   d. Find a formula for and determine the domain of $(g \circ f)(x)$
   e. Find a formula for and determine the domain of $(f \circ g)(x)$

8. Consider the linear function $f(x) = \frac{1}{3}x - 2$
   a. Find a formula for the inverse function.
   b. Make a graph for $y = f(x)$ and $y = f^{-1}(x)$ together showing symmetry through the line $y = x$.

Math 5 – Trigonometry –Spring '11 – Chapter 2 Test Solutions

1. Compute and simplify the average rate of change of $f(x)=\dfrac{1}{x-2}$ over the interval [0, *h*]. *Hint: Recall that the average rate of change over [a,b] is the slope of the line from [a, f(a)] to [b, f(b)].*

$$\frac{f(h)-f(0)}{h}=\frac{\frac{1}{h-2}-\frac{1}{-2}}{h}=\frac{1}{h}\cdot\frac{2+(h-2)}{2(h-2)}=\frac{1}{h}\cdot\frac{h}{2(h-2)}=\frac{1}{2(x-2)}$$

2. Consider the quadratic $f(x)=3x^2-6x+2$

   a. Express the quadratic function in standard (vertex) form: $y=a(x-h)^2+k$

   SOLN: $f(x)=3x^2-6x+2=3(x^2-2x)+2=3(x^2-2x+1)+2-3=\boxed{3(x-1)^2-1}$

   b. Find the coordinates of the *x*-intercepts.

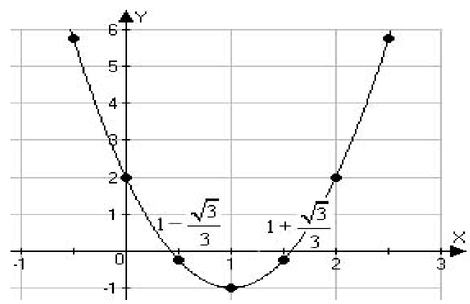
   SOLN: $y=0\Leftrightarrow 3(x-1)^2-1=0\Leftrightarrow (x-1)^2=\frac{1}{3}\Leftrightarrow x-1=\frac{\pm\sqrt{3}}{3}\Leftrightarrow \boxed{x=1\pm\frac{\sqrt{3}}{3}}$

   c. Express the quadratic function in factored form: $y=a(x-r_1)(x-r_2)$

   SOLN: $y=3\left(x-\left(1-\frac{\sqrt{3}}{3}\right)\right)\left(x-\left(1+\frac{\sqrt{3}}{3}\right)\right)=3\left(x-1+\frac{\sqrt{3}}{3}\right)\left(x-1-\frac{\sqrt{3}}{3}\right)$

   d. Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.
   SOLN: A table of values is always helpful:

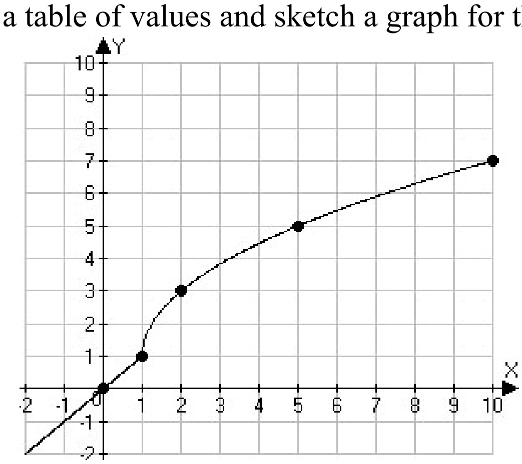
x	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
y	$\frac{23}{4}$	2	$-\frac{1}{4}$	-1	$-\frac{1}{4}$	2	$\frac{23}{4}$



3. Consider the piecewise defined function $f(x)=\begin{cases} x & x<1 \\ 1+2\sqrt{x-1} & \text{if } x\ge 1\end{cases}$

   a. Make a table of values and sketch a graph for the function.
   SOLN:

x	y
0	0
1	1
2	3
5	5
10	7

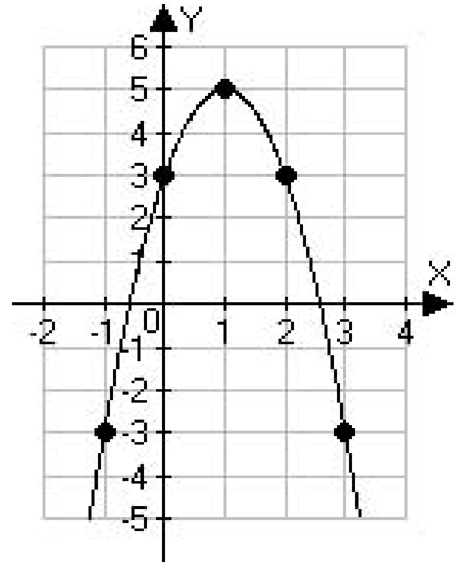


   b. Show that *f* is one-to-one and find a formula for the inverse function.
   SOLN: Since $I(x) = x$ is the identity function, it is its own inverse. For the other part, solving for x gives $x = 1+(y-1)^2/4$ so

$$f^{-1}(x)=\begin{cases} x & \text{if } x<1 \\ \frac{1}{4}(x-1)^2+1 & \text{if } x\ge 1\end{cases}$$

4. Consider the quadratic $f(x) = -2x^2 + 4x + 3$
   a. Express the quadratic function in standard form.
      SOLN: $f(x) = -2(x-1)^2 + 5 \Leftrightarrow y - 5 = -2(x-1)^2$
   b. Sketch its graph showing the position of the vertex. SOLN: (at right)
   c. What sequence of
      (i) vertical shift, SOLN: Shift down 5 by ($y \leftarrow y + 5$)
      (ii) reflection, SOLN: Reflect across x-axis by ($y \leftarrow -y$)
      (iii) vertical shrink, SOLN: Shrink vertically by ($y \leftarrow 2y$)
      (iv) horizontal shift SOLN: Shift left 5 by ($x \leftarrow x + 1$)
      In the above order, these transforms lead to the following sequence of equations:

$$\boxed{y-5=-2(x-1)^2} \to \boxed{y=-2(x-1)^2} \to \boxed{y=2(x-1)^2} \to \boxed{y=(x-1)^2} \to \boxed{y=x^2}$$



5. Given the graph of $y = f(x)$ shown at right and the given transformation, tabulate the transformed coordinate values of the key points in the graph. Then plot the given transformation on the paper at right.

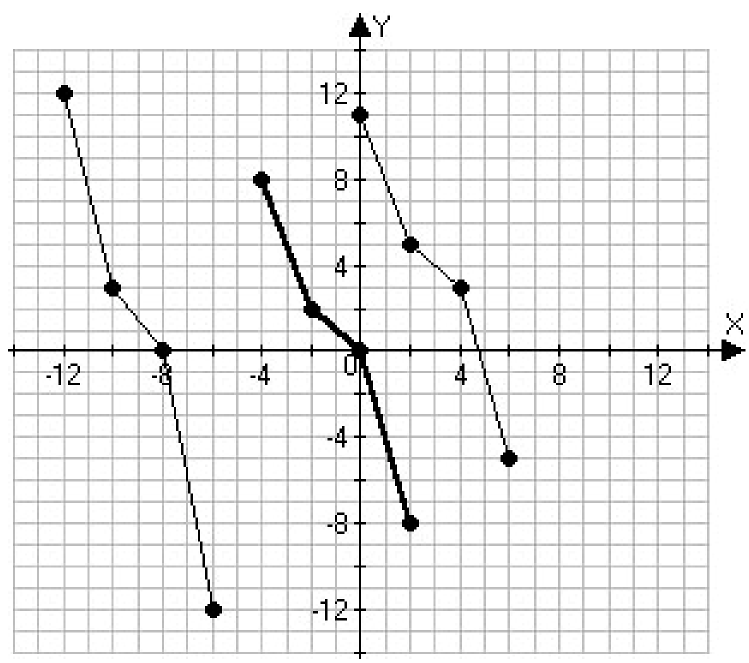
x	-4	-2	0	2
$f(x)$	8	2	0	-8

a. $y = f(x-4)+3$:

x	0	2	4	6
$y = f(x-4)+3$	11	5	3	-5

b. $y = \frac{3}{2} f(x+8)$:

x	-12	-10	-8	-6
$y = \frac{3}{2} f(x+8)$	12	3	0	-12



6. The surface area of a sphere is a function of the radius according to $S = f(r) = 4\pi r^2$ and the volume of a sphere is a function of the radius according to $V = g(r) = \frac{4}{3}\pi r^3$.

   Find a function that gives the surface area, *S*, as a function of the Volume, *V*.

   SOLN: The surface area is $V = \frac{4}{3}\pi r^3 \Leftarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so $S = f(r) = 4\pi r^2$

   $$S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3} = (4\pi)^{3/3}\left(\frac{9V^2}{16\pi^2}\right)^{1/3} = \left(36\pi V^2\right)^{1/3}$$

7. Suppose $f(x) = \sqrt{x-1}$ and $g(x) = \frac{2}{x}$.
   a. Then the domain of *f* is $[1,\infty)$.
   b. The range of *f* is $[0,\infty)$.
   c. Then the domain of *g* is $[-\infty,0)\cup[0,\infty)$.
   d. $(g \circ f)(x) = \frac{2}{\sqrt{x-1}}$ has domain $(1,\infty)$.
   e. $(f \circ g)(x) = \sqrt{\frac{2}{x}-1}$ has domain $(0,2]$.

8. If $f(x) = \frac{1}{3}x - 2$ then $f^{-1}(x) = 3x + 6$.