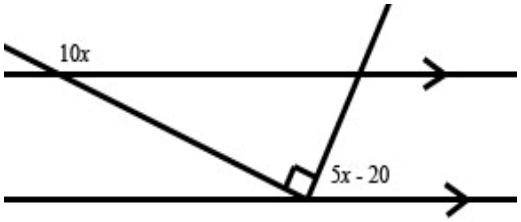
Math 5 – Trigonometry – Chapter G Test – Spring'11 Name_____

Write all responses on separate paper.

Explain your answers in detail for credit.

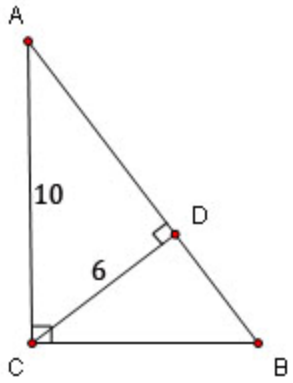
No calculators.

1. What is the degree measure of angle x in the figure at right? Explain how you know.



2. Draw an isosceles right triangle with hypotenuse = 5 cm. Label the vertices A, B, and C. Find the perimeter and area of the triangle and simplify.

3. Consider the diagram at right and assume that $\overline{AC} \perp \overline{BC}$ and that $\overline{CD} \perp \overline{AB}$.

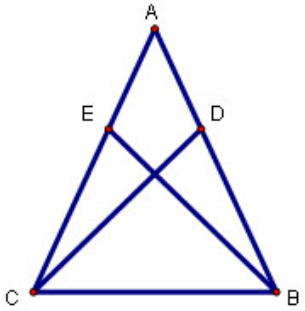


   a. Prove that $\Delta CAD \sim \Delta BCD$ are similar triangles.

   b. If $AC = 10$ and $CD = 6$, find BC
   Hint: If you find AD then you'll have the ratios $\frac{\text{hypotenuse}}{\text{short leg}}, \frac{\text{hypotenuse}}{\text{long leg}}, \frac{\text{long leg}}{\text{short leg}}$

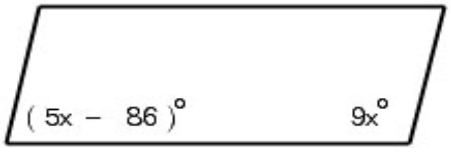
4. Draw an equilateral triangle with height = $2\sqrt{3}$ and label the vertices A, B, and C. Find the perimeter of the triangle.

5. Given the triangle shown at right, with $AC = AB$ and $\angle EBC = \angle DCB$, show that $\Delta EBA = \Delta DCA$

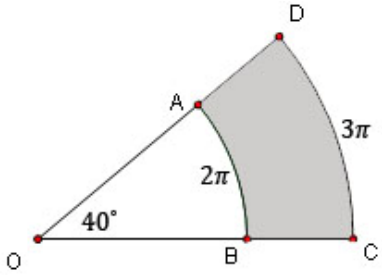


6. Draw a rhombus with diagonals of length 2 and $2\sqrt{3}$.
   a. What is the perimeter of the rhombus?
   b. What is the area of the rhombus?

7. The figure at right is a parallelogram. Find x.

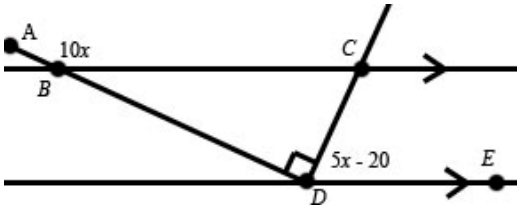


8. Consider the area of region $ABCD$ bounded by concentric arcs $AB = 2\pi$ cm and $CD = 3\pi$ cm as shown at right.
   a. Find the radii, OB and OC.
   b. Find the area of the region $ABCD$



Math 5 – Fall '08 – Chapter 1 Test Solutions

1. What is the degree measure of angle x in the figure at right? Explain how you know.



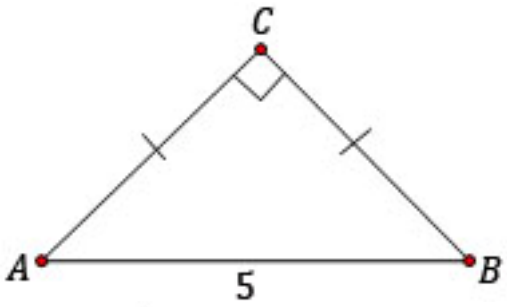
ANS: When the transversal $\overleftrightarrow{AD}$ crosses the parallels it creates equal corresponding angles, so

$\angle ABC = \angle ADE = 90^\circ + \angle CDE$

$\Leftrightarrow 10x = 90 + 5x - 20$

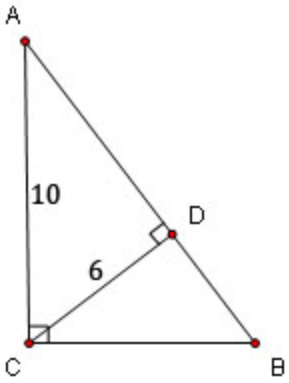
$\Leftrightarrow 5x = 70 \Leftrightarrow \boxed{x = 14^\circ}$

2. Draw an isosceles right triangle with hypot. = 5 cm. Label the vertices A, B, and C. Find the perimeter and area of the triangle and simplify.



SOLN: Let $x = AC = BC$. Then by Pythagoras' theorem, $x^2 + x^2 = 5^2$ or $x = \dfrac{5\sqrt{2}}{2}$ Thus the perimeter is $5 + 5\sqrt{2}$ cm and the area is 25/4 cm^2.

3. Consider the diagram at right and assume that $\overline{AB} \perp \overline{AC}$ and that $\overline{AD} \perp \overline{BC}$.



   a. Prove that $\Delta CAD \sim \Delta BCD$

   ANS: Since $\overline{CD} \perp \overline{AB}$, $\angle ADC = \angle BDC = 90^\circ$. Also, the two acute angles of a right triangle are complementary, so $\angle ABD + \angle ACD = 90^\circ$. Also, $\angle DBC + \angle DCB = \angle ACD + \angle DCB = 90^\circ$ and thus $\angle DBC = \angle ACD$ (angles complementary to the same angle are equal.) So by AA, the $\Delta CAD \sim \Delta BCD$

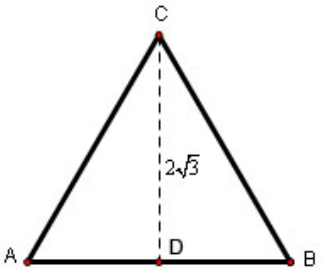
   b. If $AC = 10$ and $CD = 6$, find BC

   ANS: Since ΔCAD, we can apply the Pythagorean theorem to find $AD = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$. Now, since corresponding parts of similar triangles are proportional. Thus $\dfrac{\text{hypotenuse}}{\text{long leg}} = \dfrac{10}{8} = \dfrac{BC}{6} \Leftrightarrow BC = \dfrac{15}{2}$.

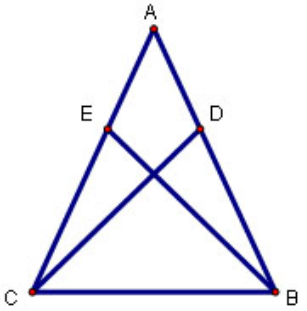
4. Draw an equilateral triangle with height = $2\sqrt{3}$ and label the vertices A, B, and C. Find the perimeter of the triangle.

ANS: By symmetry, $AD = DB$ and height CD is perpendicular to AB so $AC = AB = 2AD$ and by the Pythagoras' theorem, $AD^2 + \left(2\sqrt{3}\right)^2 = AC^2 = (2AD)^2$

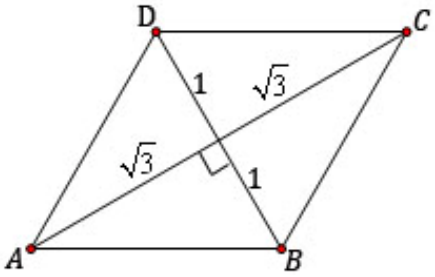
$\Leftrightarrow AD^2 + 12 = 4AD^2 \Leftrightarrow AD = 2$ and $AC = 4$. Thus the perimeter is 12.



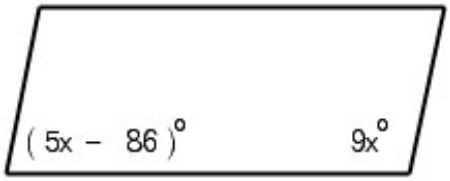
5. Given the triangle shown at right, with $AC = AB$ and $\angle EBC = \angle DCB$, show that $\Delta EBA = \Delta DCA$
ANS: The base angles of an isosceles triangle are equal, $\angle ABC = \angle ACB$. Together with the reflexive $CB = BC$ and the given $\angle EBC = \angle DCB$, we have by ASA that $\Delta EBC \cong \Delta DCB$. Thus by CPCTC, $CD = BE$. Now differences of equals are equal, so $\angle ACD = \angle ABE$, whence $\Delta EBA = \Delta DCA$ follows by SAS.



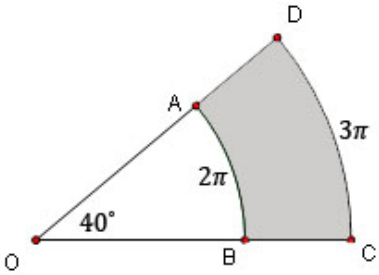
6. Draw a rhombus with diagonals of length 2 and $2\sqrt{3}$.
   a. What is the perimeter of the rhombus?
   SOLN: Since the diagonals of the rhombus are perpendicular bisectors of one another, we see the interior of the rhombus is composed of 4 congruent 30º-60º-90º triangles each with hypotenuse 2. Thus the perimeter is 8 units.
   b. What is the area of the rhombus? Area = $2\sqrt{3}$ sq. units



7. The figure at right is a parallelogram. Find x.
ANS: These angles are supplementary, so
$5x - 86 + 9x = 180 \Leftrightarrow 14x = 266 \Leftrightarrow x = 19^\circ$



8. Consider the area of region $ABCD$ bounded by concentric arcs $AB = 2\pi$ cm and $CD = 3\pi$ cm as shown at right.
   a. Find the radii, OB and OC.
   SOLN: Let R = OC and r = OB. Then
   $$3\pi = \frac{40}{360}(2\pi R) \Leftrightarrow R = \frac{27}{2} \quad 2\pi = \frac{40}{360}(2\pi r) \Leftrightarrow r = 9$$
   b. Find the area of the region $ABCD$
   SOLN: Area = big sector – small sector =



$$\frac{40}{360}(\pi R^2) - \frac{40}{360}(\pi r^2) = \frac{\pi}{9}(R^2 - r^2) = \frac{\pi}{9}(R+r)(R-r) = \frac{\pi}{9}\left(\frac{27}{2}+9\right)\left(\frac{27}{2}-9\right) = \frac{\pi}{9}\left(\frac{45}{2}\right)\left(\frac{9}{2}\right) = \frac{45\pi}{4}$$