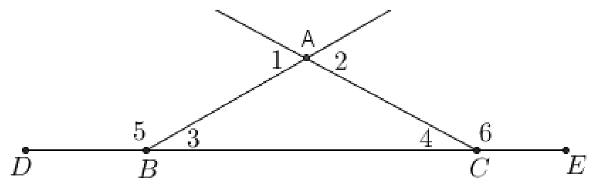
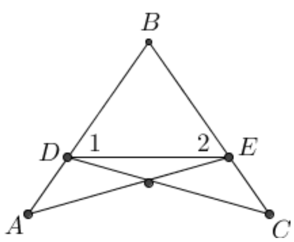
Math 5 - Fall '16 - Test 1 Solutions

1. For each of the following, let the two angles be represented by A and B. Obtain two equations for each case, and then solve the system to find the angles.
   (a) The angles are adjacent and form an angle measuring $100°$. Their difference is $22°$.
   Sol'n: $A + B = 100$ and $A = B + 22$ Substituting, we have $(B + 22) + B = 100 \Leftrightarrow 2B = 78 \Leftrightarrow$ $\boxed{B{=}39}$ and so $\boxed{A{=}61}$
   (b) The angles are complementary. One measures $10°$ more than three times the other.
   Sol'n: $A + B = 90$ and $A = 3B + 10$ Substituting, we have $(3B + 10) + B = 90 \Leftrightarrow 4B = 80 \Leftrightarrow$ $\boxed{B{=}20}$ and so $\boxed{A{=}70}$
   (c) The angles are supplementary. One measures $10°$ more than four times the other. **Sol'n:** $A+B = 180$ and $A = 4B + 10$ Substituting, we have $(4B + 10) + B = 180 \Leftrightarrow 5B = 170 \Leftrightarrow$ $\boxed{B{=}34}$ and so $\boxed{A{=}146}$

2. Answer each of the following by stating the basic angle theorem needed.
   (a) Why does $m\angle 1 = m\angle 2$?
   Sol'n: The angles have the same measure because vertical angles are congruent.
   (b) Why does $m\angle DBC = m\angle ECB$?
   Sol'n: Those are both straight angles and all straight angles have a measure of $180°$
   (c) If $m\angle 3 = m\angle 4$, why does $m\angle 5 = m\angle 6$?
   Sol'n: Angles supplementary to congruent angles are congruent.



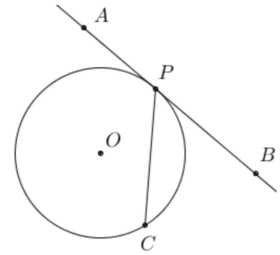
3. 

Given:
$\angle 1 \cong \angle 2$.
$\overline{AD} \cong \overline{EC}$.

Prove:
$\triangle ABE \cong \triangle CBD$

Statement	Reason
1. $\overline{BD} \cong \overline{BE}$	1. If two angles of a $\triangle$ are $\cong$ then the sides opposite are $\cong$
2. $\overline{BD} + \overline{DA} = \overline{BA}$	2.
3. $\overline{BE} + \overline{EC} = \overline{BC}$	3. The whole is the sum of its parts.
4. $\overline{BA} = \overline{BC}$.	4. Things equal to equal things are equal to eachother.
5. $\angle B \cong \angle B$	5. Reflexive postulate for congruence.
6. $\triangle ABE \cong \triangle CBD$	6. SAS

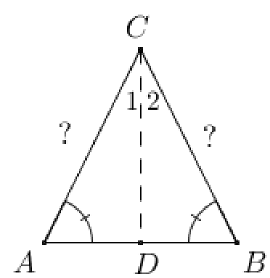
4. **Given**:
(0) with tangent $\overleftrightarrow{AB}$ at P.
Chord $\overline{PC}$
Prove:
$\angle BPC = \frac{1}{2}\widehat{PC}$



Statement	Reason
1. Draw chord $\overline{CD}$ parallel to $\overleftrightarrow{AB}$.	1. Parallel postualte..
2. $\widehat{PC} \cong \widehat{PD}$	2. Parallel lines cut off $\cong$ arcs in a circle.
3. $\overline{DP} = \overline{CP}$	3. Arcs are $\cong$ $\Leftrightarrow$ corresponding chords are $\cong$.
4. $\angle PDC = \frac{1}{2}\widehat{PC}$	4. An inscribed $\angle = \frac{1}{2}$ the intercepted arc.
5. $\angle PDC \cong \angle PCD$.	5. The base angles of an isosceles $\triangle$ are $\cong$.
6. $\angle BPC \cong \angle PCD$.	6. Trans. $\overline{PC}$ cuts $\overline{AB} \parallel \overline{DC}$, alt. int. $\angle$s are $\cong$
7. $\angle BPC = \frac{1}{2}\widehat{PC}$.	7. Things = to the same thing are = to eachother.

5. Write a two-column proof for the statement: "If two angles of a triangle are congruent then the triangle is isosceles.

Given: $\triangle ABC$ with $\angle A \cong \angle B$.
Prove:$\overline{AC} \cong \overline{BC}$



Statement	Reason
1. Draw $\overline{CD}$ bisecting $\angle C$.	1. Postulate: Every angle can be bisected.
2. $\angle 1 \cong \angle 2$	2. To bisect is to divide into two $\cong$ parts.
3. $\angle A \cong \angle B$	3. Given.
4. $\overline{CD} \cong \overline{CD}$.	4. Reflexive postulate for congruence.
5. $\triangle ACD \cong \triangle BCD$.	5. AAS
6. $\overline{AC} \cong \overline{BC}$.	6. CPCTC