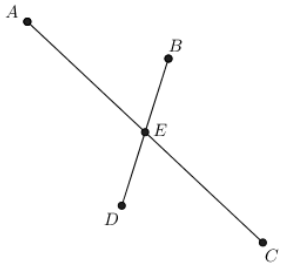
Math 5 - Trigonometry – Final Exam Solutions

1.



Given:

$\overline{AC}$ and $\overline{BD}$ intersect at E which is the midpoint of both $\overline{AC}$ and $\overline{BD}$.

Prove:

$ABCD$ is a parallelogram

Statement	Reason
1. $\overline{AE} \cong \overline{EC}$ and $\overline{BE} \cong \overline{ED}$	1. Definition of midpoint.
2. $\angle AEB \cong \angle DEC$ and $\angle AED \cong \angle BEC$	2. Vertical angles are congruent.
3. Draw AB, BC, CD and DA.	3. You can draw a segment between any two points.
4. $\triangle ABE \cong \triangle CDE$ & $\triangle BCE \cong \triangle DAE$	4. SAS
5. $\angle DBC \cong \angle BDA$ and $\angle DBA \cong \angle BDC$	5. Corresponding parts of $\cong$ triangles are $\cong$
6. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$	6. A transversal making alt. int, $\angle$s $\cong$, $\Rightarrow$ lines $\parallel$
7. $ABCD$ is a parallelogram	7. By definition of parallelogram.

2. Let $f(x) = \dfrac{\sqrt{x-1}}{x-3}$.

   (a) Evaluate $f(1)$, $f(5)$ and $f(a^2+1)$

   ANS: $f(1) = 0, f(5) = \dfrac{\sqrt{4}}{2} = 1, f(a^2+1) = \dfrac{|a|}{a^2-2}$

   (b) Find the domain of f.
   ANS: $[1,3) \cup (3,\infty)$

   (c) What is the average rate of change of f between $x = 5$ and $x = 17$?
   ANS: $\dfrac{f(17)-f(5)}{17-5} = \dfrac{\frac{4}{14}-1}{12} = \dfrac{-5}{7} \cdot \dfrac{1}{12} = -\dfrac{5}{84}$

3. Let $f(x) = \sqrt{4-x}$.

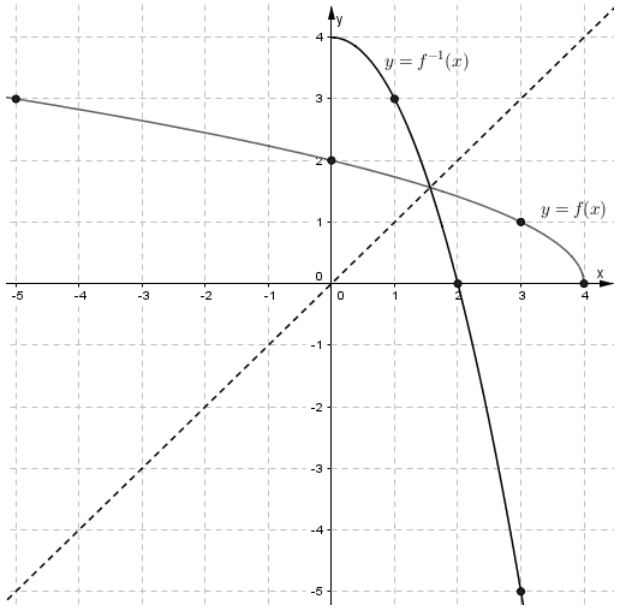
   (a) Find a formula for the inverse function, $f^{-1}(x)$.
   Ans: $y = \sqrt{4-x} \Leftrightarrow y^2 = 4 - x$ and $y \geq 0 \Leftrightarrow x = 4 - y^2$ and $y \geq 0$. So the inverse function can be specified at $y = 4 - x^2$ and $x \geq 0$

   (b) Find the domain of f^{-1}. Recall that the range of f is the domain of f^{-1}.
   As mentioned in part (a), the domain is $x \geq 0 = [0,\infty)$

   (c) Complete the table for $f^{-1}(x)$:

x	0	1	2	3
$f^{-1}(x)$	4	3	0	-5

(d) Sketch graphs for $y = f(x)$ and $y = f^{-1}(x)$ together showing symmetry through the line $y = x$:



4. Find the exact value of each expression:

(a) $\sin\dfrac{4\pi}{3} = -\dfrac{\sqrt{3}}{2}$ (b) $\tan\dfrac{7\pi}{6} = \dfrac{\sqrt{3}}{3}$ (c) $\sec\dfrac{13\pi}{6} = \dfrac{2\sqrt{3}}{3}$ (d) $\cos\dfrac{17\pi}{3} = \dfrac{1}{2}$

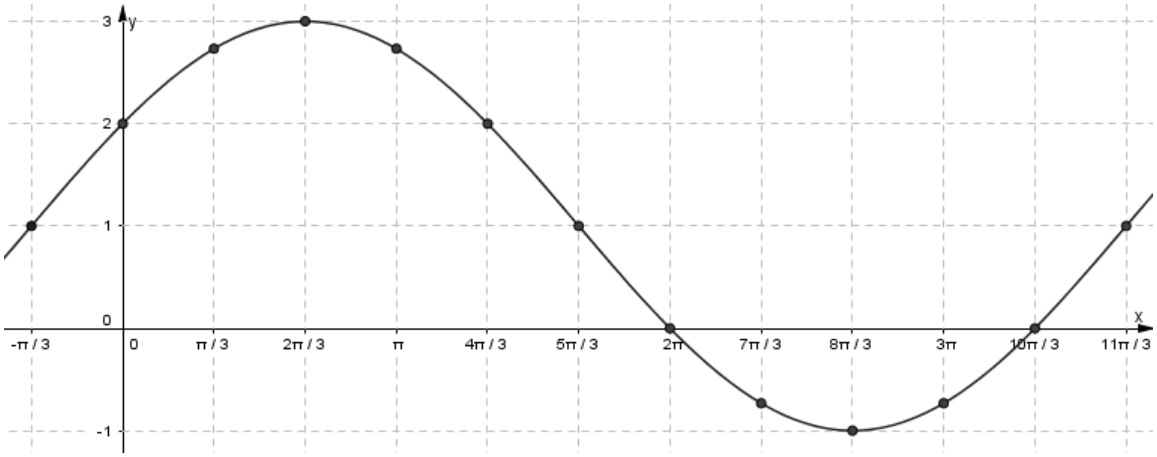
5. Let $f(x) = 1 + 2\sin\left(\frac{1}{2}x + \dfrac{\pi}{6}\right)$

(a) Complete the table:

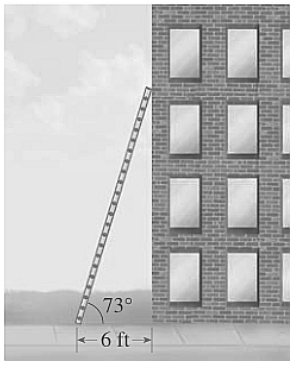
x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
$f(x)$	1	2	$1+\sqrt{3}$	3	$1+\sqrt{3}$	2	1

(b) What are the amplitude, phaseshift, period and line of equilibrium for f ?
Amplitude $= 2$, phaseshift $= -\dfrac{\pi}{3}$, period $= 4\pi$ and $y = 1$ is the equilibrium.

(c) Sketch a graph showing one complete oscillation for f.



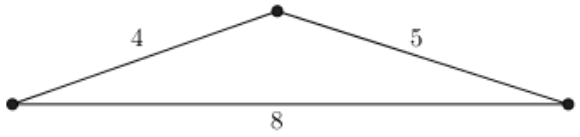
6. The base of the ladder in the figure is 6 ft from the building, and the angle formed by the ladder and the ground is 73°.



   (a) How high up the building does the ladder touch?
   ANS: Let H = height up the building the ladder reaches.
   Then $\tan 73° = H/6 \Leftrightarrow H = 6\tan 73° \approx 19.6$ ft.

   (b) What is the length of the ladder?
   ANS: Let L = length of the ladder.
   Then $\cos 73° = 6/L \Leftrightarrow L = 6\sec 73° \approx 20.5$ ft.

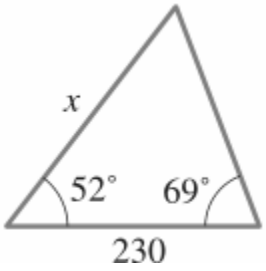
7. Refer to the figure below.



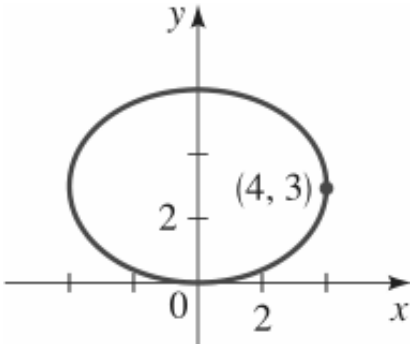
   (a) Find the angle opposite the longest side.
   This is a SSS situation, so we use the law of cosines:
   $\cos\theta = \dfrac{+16+25-64}{2(4)(5)} = -\dfrac{23}{40} \Leftrightarrow \theta = \arccos\dfrac{-23}{40} \approx 125.1°$.

   (b) Find the area of the triangle.
   ANS: Area $= \frac{1}{2}(4)(5)\sin(\arccos(-23/40)) = 10\sin(\arccos(23/40) = \dfrac{\sqrt{1071}}{4} \approx 8.18$ square units. Alternatively, Heron's formula with semiperimeter of $\dfrac{17}{2}$ gives area $= \sqrt{\dfrac{17}{2}\cdot\dfrac{9}{2}\cdot\dfrac{7}{2}\cdot\dfrac{1}{2}} = \dfrac{\sqrt{1071}}{4}$, too.

8. Find the side labeled x.
   Use the fact that the sum of interior angles is 180° and law of sines:
   $\dfrac{x}{\sin 69°} = \dfrac{230}{\sin 59°} \Leftrightarrow x = \dfrac{230\sin 69°}{\sin 59°} \approx 250.5$



9. Consider the ellipse whose graph is shown.



   (a) What are the coordinates of the center?
   $(0, 3)$

   (b) Find an equation for the ellipse.
   $\dfrac{x^2}{16} + \dfrac{(y-3)^2}{9} = 1$

   (c) What are the coordinates of the foci?
   $b^2 = 16 - 9 = 7$ so the foci are at $(\pm\sqrt{7}, 3)$

10. Consider the hyperbola whose equation is $(x-1)^2 - y^2 = 1$

(a) Find the coordinates of center.
ANS: $(1, 0)$

(b) Find the x-intercepts of the hyperbola.
ANS: If $y = 0$ then $(x-1)^2 = 1 \Leftrightarrow x = 1 \pm 1$, so $(0, 0)$ and $(2, 0)$ are the $x-$intercepts.

(c) Find the coordinates of the two foci.
$c^2 = a^2 + b^2 = 1 + 1 = 2$ so the foci are at $(1 \pm \sqrt{2}, 0)$

(d) Find equations for the asymptotes of the hyperbola.
$y = x - 1$ and $y = -x + 1$

(e) Sketch a graph for the hyperbola