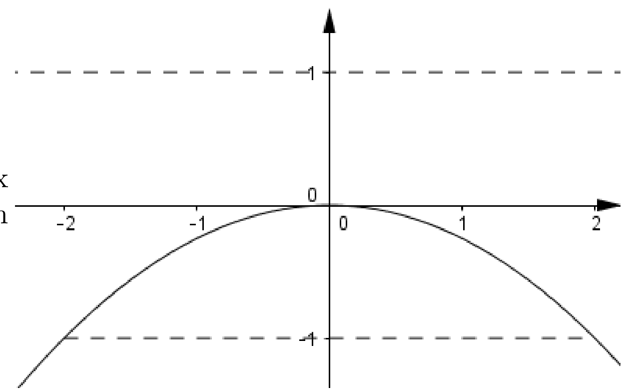
Exam 5: Chapter 11 Solutions

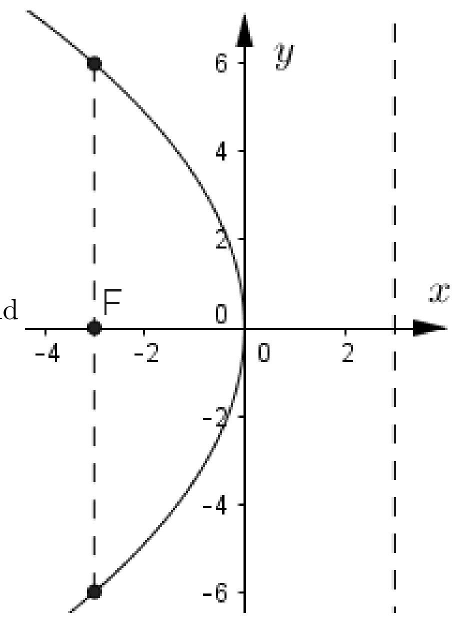
Write all responses on separate paper. Remember to organize your work clearly. You may *not* use your books, notes on this exam, but you will need a scientific calculator to do some approximations.

1. (24 points) Find an equation for th eparabola that satisfies the given conditions. In each case, sketch a graph and draw the line segment which is the focal diameter.

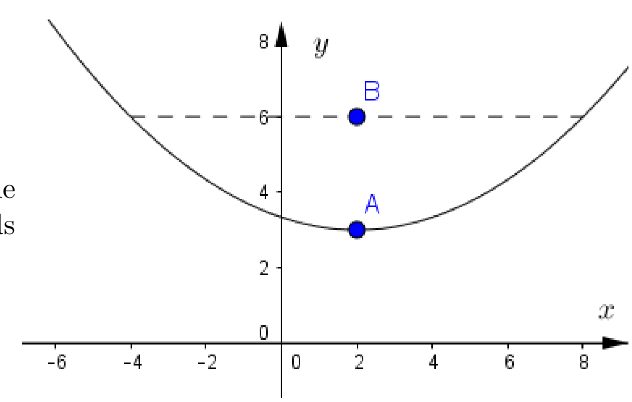
   (a) Vertex at $(0, 0)$ and focus at $(0, -1)$
   ANS: The equation is $-4y = x^2$. The directrix is $y = 1$ and the focal diameter extends from $(-2, -1)$ to $(2, -1)$.



   (b) Vertex at $(0, 0)$ and directrix $x = 3$
   ANS: The equation is $-12x = y^2$. The focus is at $(-3, 0)$ and the focal diameter extends from $(-3, -6)$ to $(-3, 6)$.

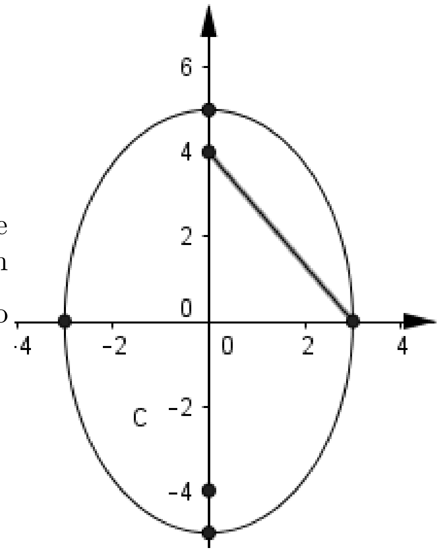


   (c) Vertex at $(2, 3)$ and directrix $y = 0$
   ANS: The equation is $12(y - 3) = (x - 2)^2$. The focus is at $(2, 6)$ and the focal diameter extends from $(-4, 6)$ to $(8, 6)$.

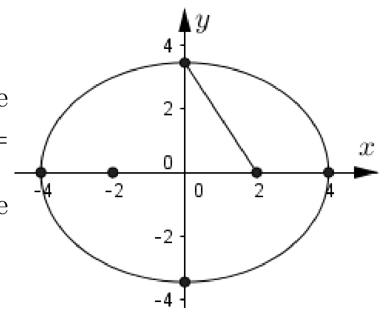


2. (24 points) Find an equation for the ellipse that satisfies the given conditions. In each case, sketch a graph and draw the line segment from an end of the minor axis to a focus (the length of this segment should be a).

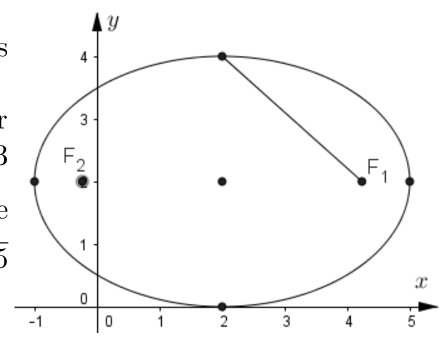
(a) Vertices at $(0, \pm 5)$ and foci at $(0, \pm 4)$
ANS: From the location of the vertices and the focus we have $a = 5$ and $c = 4$ so $b^2 = a^2 - c^2 = 25 - 16 = 9$ and the equation of the ellipse is $\dfrac{y^2}{25} + \dfrac{x^2}{9} = 1$. The line segment from $(4, 0)$ to $(0, 3)$ has length 5.



(b) Foci at $(\pm 2, 0)$ and eccentricity $= \frac{1}{2}$
ANS: From the location of the foci we have $c = 2$ and since the eccentricity is $e = \frac{c}{a} = \frac{1}{2}$ we have $\dfrac{2}{a} = \dfrac{1}{2} \Leftrightarrow a = 4$ and so $b^2 = a^2 - c^2 = 16 - 4 = 12$ Thus the equation is $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$ and a line segment from $(0, 2\sqrt{3})$ to $(2, 0)$ has length 4.

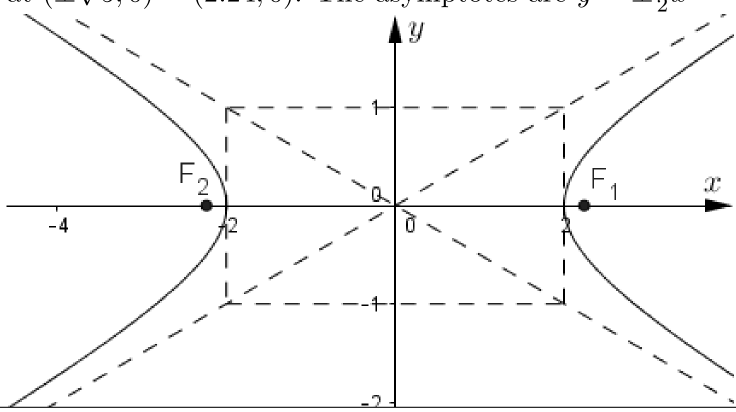


(c) Endpoints of the minor axis at $(2, 0)$ and $(2, 4)$ and endpoints of the major axis at $(-1, 2)$ and $5, 2)$
ANS: The center of the ellipse is the midpoint of the minor axis and the major axis, at $(2, 2)$ and thus we see that $a = 3$ and $b = 2$ and the equation is $\dfrac{(x-2)^2}{9} + \dfrac{(y-2)^2}{4} = 1$. The foci are at $(2, 2 \pm c)$ where $c^2 = a^2 - b^2 = 9 - 4 = 5 \Leftrightarrow c = \sqrt{5}$ and the line segment from $(0, 4)$ to $(2 + \sqrt{5}, 2)$ has length 3.

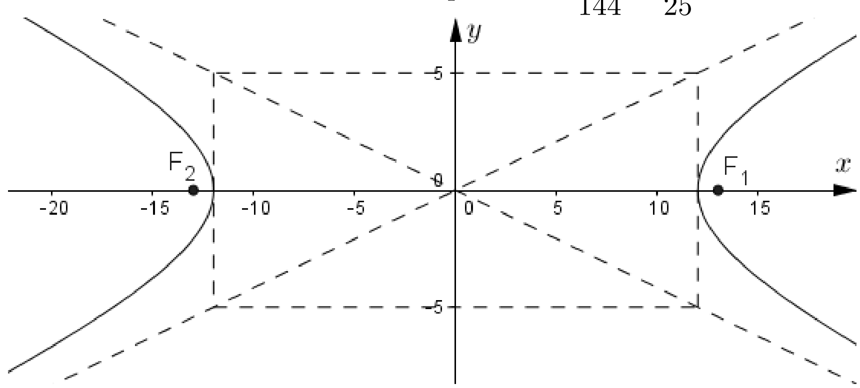


3. (20 points) Find an equation for the hyperbola that satisfies the given conditions. In each case, find the equations of the asymptotes and sketch a graph showing the hyperbola with its vertice, foci an asymptotes.

   (a) Equation is $\dfrac{x^2}{4} - y^2 = 1$ ANS: Evidently, $a = 2$ and $b = 1$ so $c^2 = a^2 + b^2 = 5$ and the foci are at $(\pm\sqrt{5}, 0) \approx (2.24, 0)$. The asymptotes are $y = \pm\frac{1}{2}x$



   (b) Vertices at $(\pm 12, 0)$ and foci at $(\pm 13, 0)$
   ANS: From the location of the vertices and foci we have $a = 12$ and $c = 13$ so $b^2 = c^2 - a^2 = 169 - 144 = 25$ so $b = 5$ and the equation is $\dfrac{x^2}{144} - \dfrac{y^2}{25} = 1$ and the asymptotes are $y = \pm\dfrac{5}{12}$.



4. (20 points) Each equation is either an ellipse or a hyperbola. Complete the squares to write the equation in standard form and then identify it as either an ellipse or a hyperbola and state the coordinates of its center.

   (a) $x^2 + 2y^2 - 4y = 0$
   ANS: $x^2 + 2(y^2 - 2y) = 0 \Leftrightarrow x^2 + 2(y^2 - 2y + 1) = 2 \Leftrightarrow x^2 + 2(y-1)^2 = 2 \Leftrightarrow \dfrac{x^2}{2} + (y-1)^2 = 1$ is the equation for an ellipse centered $(0, 1)$.

   (b) $x^2 - 8x = y^2 - 6y \Leftrightarrow x^2 - 8x + 16 + 9 = y^2 - 6y + 9 + 16 \Leftrightarrow (x-4)^2 + 9 = (y-3)^2 + 16 \Leftrightarrow (x-4)^2 - (y-3)^2 = 7 \Leftrightarrow \dfrac{(x-4)^2}{7} - \dfrac{(y-3)^2}{7} = 1$ is a hyperbola centered at $(4, 3)$

5. (12 points) Use the Pythagorean identity to find parametric equations for each given conic.

   (a) $x^2 + \dfrac{(y-3)^2}{4} = 1$
   ANS: $x = \cos(t)$ and $y = 3 + 2\sin(t)$

   (b) $(y-2)^2 - x^2 = 1$
   ANS: $x = \tan(t)$ and $y = 2 + \sec(t)$