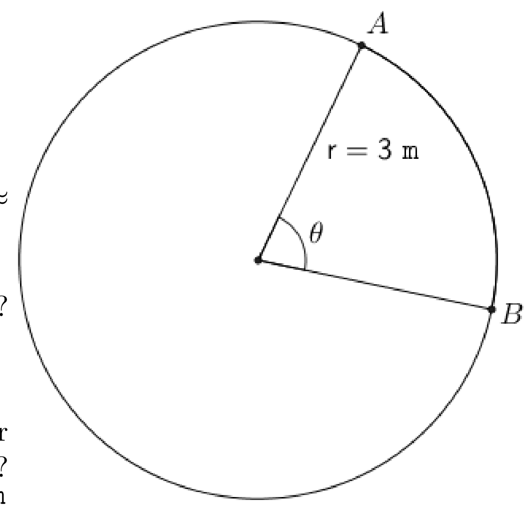
Exam 4: Chapter 6 Solutions

Write all responses on separate paper. Remember to organize your work clearly. You may *not* use your books, notes on this exam, but you will need a scientific calculator to do some approximations.

1. (28 points) In the circle at right, arc length $\widehat{AB} = 4$ meters subtends a central angle of θ.



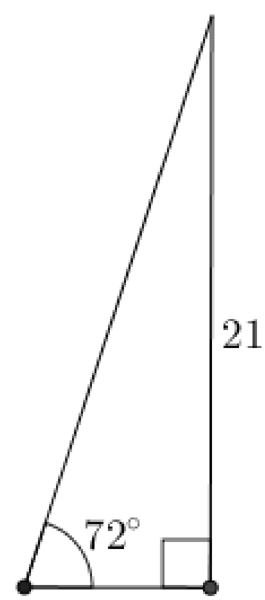
   (a) What is the radian measure of θ?
   ANS: By definition, $\theta = \dfrac{\texttt{arclength}}{\texttt{radius}} = \dfrac{4}{3}$

   (b) What is the degree measure of θ? $\dfrac{4}{3} \cdot \dfrac{180^\circ}{\pi} = \left(\dfrac{240}{\pi}\right)^\circ \approx 76.6944^\circ$

   (c) What is the area of a sector with central angle θ in this circle?
   ANS: Area $= \dfrac{r^2\theta}{2} = \dfrac{9 \cdot \frac{4}{3}}{2} = 6$ square units.

   (d) If A is rotating around the circle at a rate of 45 rotations per minute, what is the linear speed of A in kilometers per hour?
   $45\dfrac{\texttt{rev}}{\texttt{min}} \cdot \dfrac{2\pi\texttt{radians}}{\texttt{rev}} \cdot 3\texttt{m} \cdot \dfrac{60\texttt{min}}{\texttt{hr}} \cdot \dfrac{1\texttt{km}}{1000\texttt{m}} = 16.2\pi\dfrac{\texttt{km}}{\texttt{hr}} \approx 50.9\dfrac{\texttt{km}}{\texttt{hr}}$

2. (24 points) Refer to the triangle at right to answer the following questions.



   (a) Find the length of the leg adjacent to the given angle. Approximate your answer to the nearest thousandth.
   ANS: $\tan(72^\circ) = \dfrac{21}{\texttt{ADJ}} \Leftrightarrow \texttt{ADJ} = \dfrac{21}{\tan(72^\circ)} \approx 6.823$

   (b) Find the length of the hypotenuse. Approximate your answer to the nearest thousandth. ANS: $\sin(72^\circ) = \dfrac{21}{\texttt{HYP}} \Leftrightarrow \texttt{HYP} = \dfrac{21}{\sin(72^\circ)} \approx 22.081$

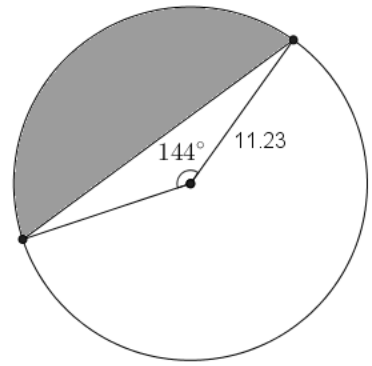
   (c) Find the radian measure of the other acute angle.
   ANS: $18^\circ \cdot \dfrac{\pi}{180^\circ} = \dfrac{\pi}{10}$

3. (14 points) A laser beam is to be directed toward the center of the moon, but the beam strays 0.1° from its intended path.

   (a) How far has the beam diverged from its assigned target when it reaches the moon? (The distance from the earth to the moon is 380,000 km. Round your answer to the nearest kilometer.)
   ANS: The arc length along the circular orbit of the moon induced by a central angle of 0.1° is $0.1^\circ \cdot \dfrac{\pi}{180^\circ} \cdot 380,000\texttt{km} \approx 663\texttt{km}$

   (b) The radius of the moon is about 1700 km. Will the beam strike the moon?
   ANS: Yes!

4. (10 points) Approximate the area of the shaded region in the figure at right to the nearest hundredth.



ANS: `Area` = `area of sector` − `area of triangle` =

$= \frac{1}{2}144° \cdot \frac{\pi}{180°} \cdot (11.23)^2 - \frac{1}{2}(11.23)^2 \sin\left(\frac{4\pi}{5}\right) =$ (may omit next)

$= \left(\frac{2\pi}{5} - \frac{\sqrt{10-2\sqrt{5}}}{8}\right) \cdot 126.1129 \approx 121.414$

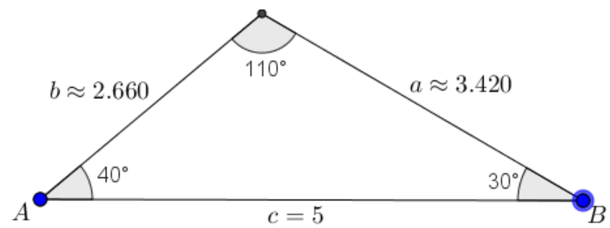
5. (24 points) Solve each triangle given below. That is, find all side lengths and interior angles.

(a) (ASA case) $\angle A = 40°, B = 30°$ and the included side, $c = 5$. $\boxed{\angle C = 180° - 30° - 40° = 110°}$

By the law of sines, $\frac{a}{\sin 40°} = \frac{5}{\sin 110°}$

$\Leftrightarrow \boxed{a = \frac{5\sin 40°}{\sin 110°} \approx 3.420}$.

Similarly, $\boxed{b = \frac{5\sin 30°}{\sin 110°} \approx 2.660}$



(b) (ASS case) $\angle A = 30°$, $b = 5, a = 7$

Since $a > b$ there is only one solution.

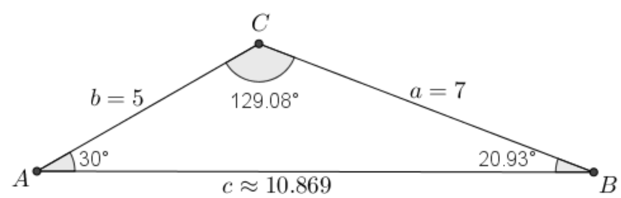
By law of sines, $\sin(B) = \frac{5 \cdot \sin(30°)}{7} = \frac{5}{14} \Leftrightarrow$

$\boxed{B = \arcsin\left(\frac{5}{14}\right) \approx 20.925°}$.

Since the sum of interior angles is 180°,

$\boxed{\angle C \approx 180° - 30° - 20.925° = 129.075°}$ and by the law of sines,

$\boxed{c \approx \frac{7\sin(129.075°)}{\sin(30°)} = 14\sin(129.075°) \approx 10.869}$



(c) (SAS case) $b = 3, c = 5, \angle A = \arcsin 0.6$

By the law of cosines, $a^2 = 9 + 25 - 30\cos(\arcsin 0.6) = 34 - 30(0.8) = 10 \Rightarrow$

$\boxed{a = \sqrt{10}}$. Now using the law of sines,

$\angle B = \arcsin\left(\frac{5\sin(\arcsin(0.6))}{\sqrt{10}}\right) = \arcsin\left(\frac{1.8}{\sqrt{10}}\right)$

$\Rightarrow \boxed{\angle B \approx 34.69515°}$

$\Rightarrow \angle C \approx 180° - 34.69515° - \arcsin(0.6)$

$\Rightarrow \angle C \approx 145.30485° - 36.86990°$

$\Rightarrow \boxed{\angle C \approx 108.43494882°}$